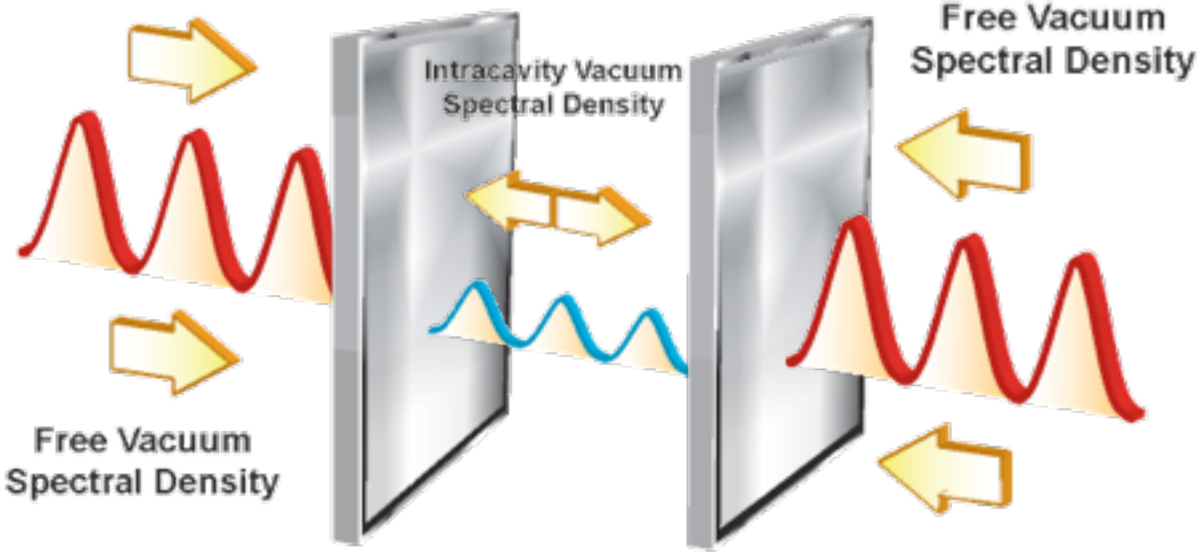
Casimir theory-experiment comparison
(and the role of patch effects)

Diego A. R. Dalvit
Theoretical Division
Los Alamos National Laboratory

Outline of this Talk

- Brief introduction to Casimir physics
  - Basic theory
  - Modern experiments
  - Lifshitz formula and scattering theory
- Theory-experiment comparison
  - Electrostatic calibration, residual force measurement
  - Comparing theory and experiment
- Electrostatic patch effects
  - Systematic effect relevant to various experiments
  - Estimating patch effects
  - Measuring patch distributions
  - What does it say for Casimir force experiments

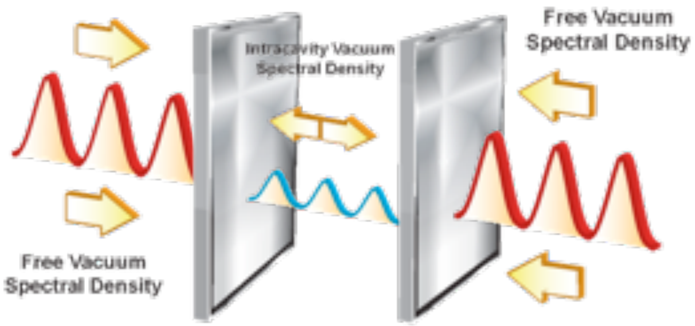
Introduction: a force from nothing



The Casimir force

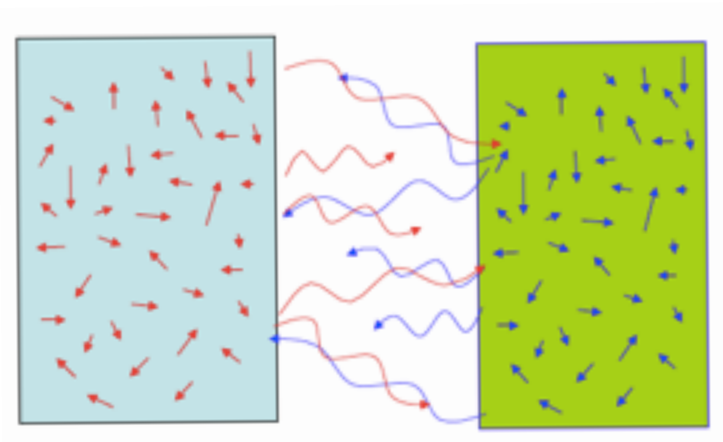
- The Casimir effect is a universal effect from confinement of vacuum fluctuations: it depends only on $\hbar$, c and geometry

$$\frac{F}{A} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4} \qquad (130\mathrm{nN/cm^2}\ @\ d = 1\mu\mathrm{m})$$



Dutch physicist H. Casimir

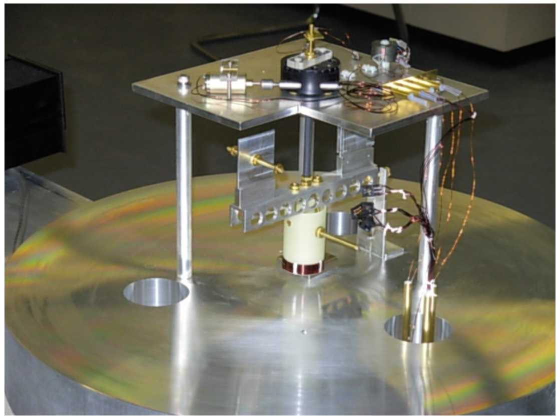
- It can also be interpreted as arising from fluctuations of charges and currents within the materials



- The magnitude and sign of the force depends on geometry, material composition, and temperature

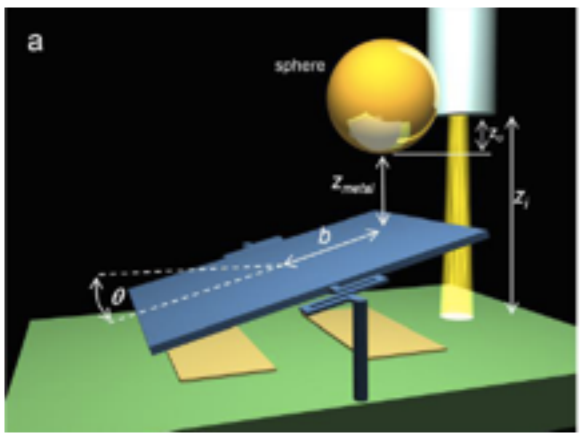
Modern experiments

- Torsion pendulum



- Atomic force microscope

- MEMS and NEMS



Lifshitz formula - Scattering theory

Lifshitz formula (1956) - Casimir interaction energy between two slabs

$$\frac{E(d)}{A} = \hbar \sum_p \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \coth\left(\frac{\hbar\omega}{2k_B T}\right) \mathrm{Im}\,\log[1 - R_{1,p}(\omega,k)\; R_{2,p}(\omega,k)\; e^{2id\sqrt{\omega^2/c^2 - k^2}}]$$

Fresnel reflection coefficients

$$R_{\mathrm{TE}} = \frac{k_z - \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}{k_z + \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}} \qquad R_{\mathrm{TM}} = \frac{\epsilon(\omega)k_z - \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}{\epsilon(\omega)k_z + \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}$$

Lifshitz formula - Scattering theory

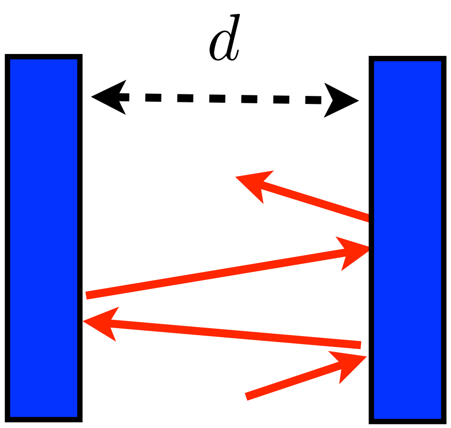
Lifshitz formula (1956) - Casimir interaction energy between two slabs

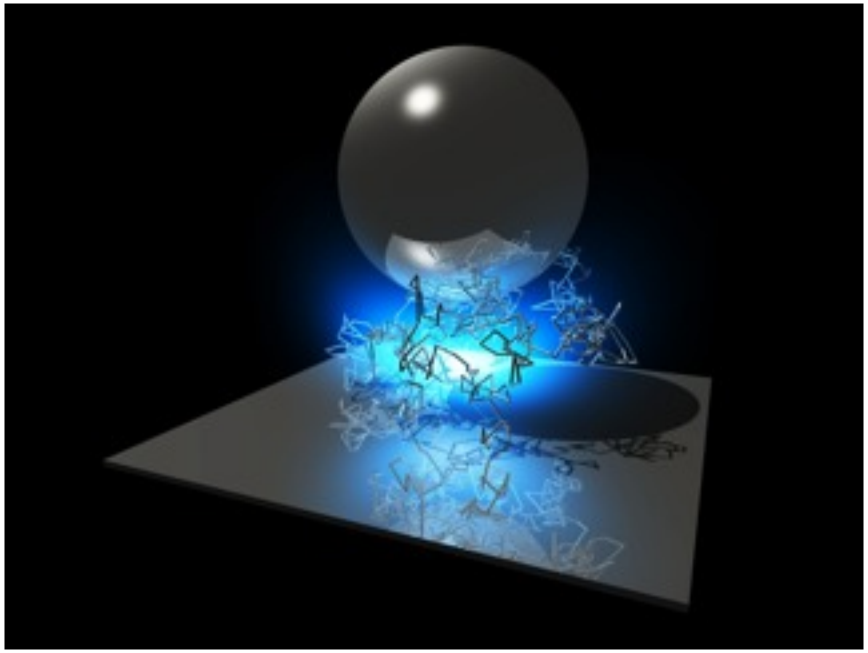
$$\frac{E(d)}{A} = \hbar \sum_p \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \coth\left(\frac{\hbar\omega}{2k_BT}\right) \mathrm{Im}\,\log[1 - R_{1,p}(\omega,k)\; R_{2,p}(\omega,k)\; e^{2id\sqrt{\omega^2/c^2-k^2}}]$$

Fresnel reflection coefficients $R_{\mathrm{TE}} = \frac{k_z - \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}{k_z + \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}$ $R_{\mathrm{TM}} = \frac{\epsilon(\omega)k_z - \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}{\epsilon(\omega)k_z + \sqrt{\epsilon(\omega)\omega^2/c^2 - k^2}}$

The log factor can be re-written as

$$\propto \sum_{n=1}^{\infty} \frac{1}{n}[R_{1,p}\; e^{idk_z}\; R_{2,p}\; e^{idk_z}]^n$$



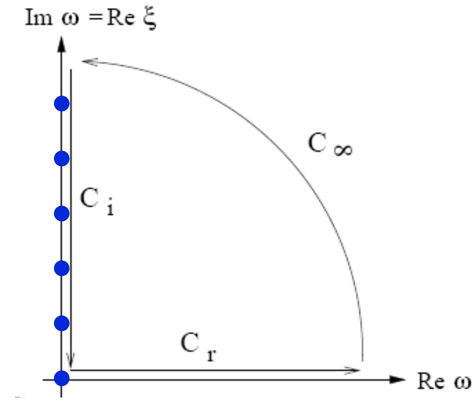


Scattering theory for Casimir effects

Going to imaginary frequencies

The function $\coth(\hbar\omega/2k_BT)$ has poles on the imaginary frequency axis at

$$\omega_m = i\xi_m \quad , \quad \xi_m = m\frac{2\pi k_B T}{\hbar}$$



After Wick rotation:

$$\boxed{\frac{F}{A} = -2k_BT\sum_p {\sum_{m=0}^{\infty}}' \int \frac{d^2\mathbf{k}}{(2\pi)^2}\sqrt{\xi_m^2/c^2+k^2}\,\frac{R_{1,p}(i\xi_m,k)R_{2,p}(i\xi_m,k)e^{-2d\sqrt{\xi_m/c^2+k^2}}}{1-R_{1,p}(i\xi_m,k)R_{2,p}(i\xi_m,k)e^{-2d\sqrt{\xi_m/c^2+k^2}}}}$$

$$\epsilon(i\xi) = 1 + \frac{2}{\pi}\int_0^{\infty}\frac{\omega\epsilon''(\omega)}{\omega^2+\xi^2}d\omega$$

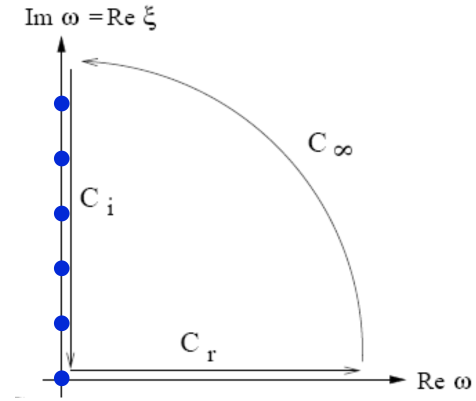
Kramers-Kronig (causality)

Casimir physics is a broad-band frequency phenomenon

Going to imaginary frequencies

The function $\coth(\hbar\omega/2k_BT)$ has poles on the imaginary frequency axis at

$$\omega_m = i\xi_m \quad , \quad \xi_m = m\frac{2\pi k_BT}{\hbar}$$



After Wick rotation:

$$\boxed{\frac{F}{A} = -2k_BT\sum_p \sum_{m=0}^{\infty'} \int \frac{d^2\mathbf{k}}{(2\pi)^2}\sqrt{\xi_m^2/c^2+k^2}\,\frac{R_{1,p}(i\xi_m,k)R_{2,p}(i\xi_m,k)e^{-2d\sqrt{\xi_m/c^2+k^2}}}{1-R_{1,p}(i\xi_m,k)R_{2,p}(i\xi_m,k)e^{-2d\sqrt{\xi_m/c^2+k^2}}}}$$

$$\epsilon(i\xi) = 1 + \frac{2}{\pi}\int_0^\infty \frac{\omega\epsilon''(\omega)}{\omega^2+\xi^2}d\omega$$

Kramers-Kronig (causality)

Casimir physics is a <u>broad-band</u> frequency phenomenon

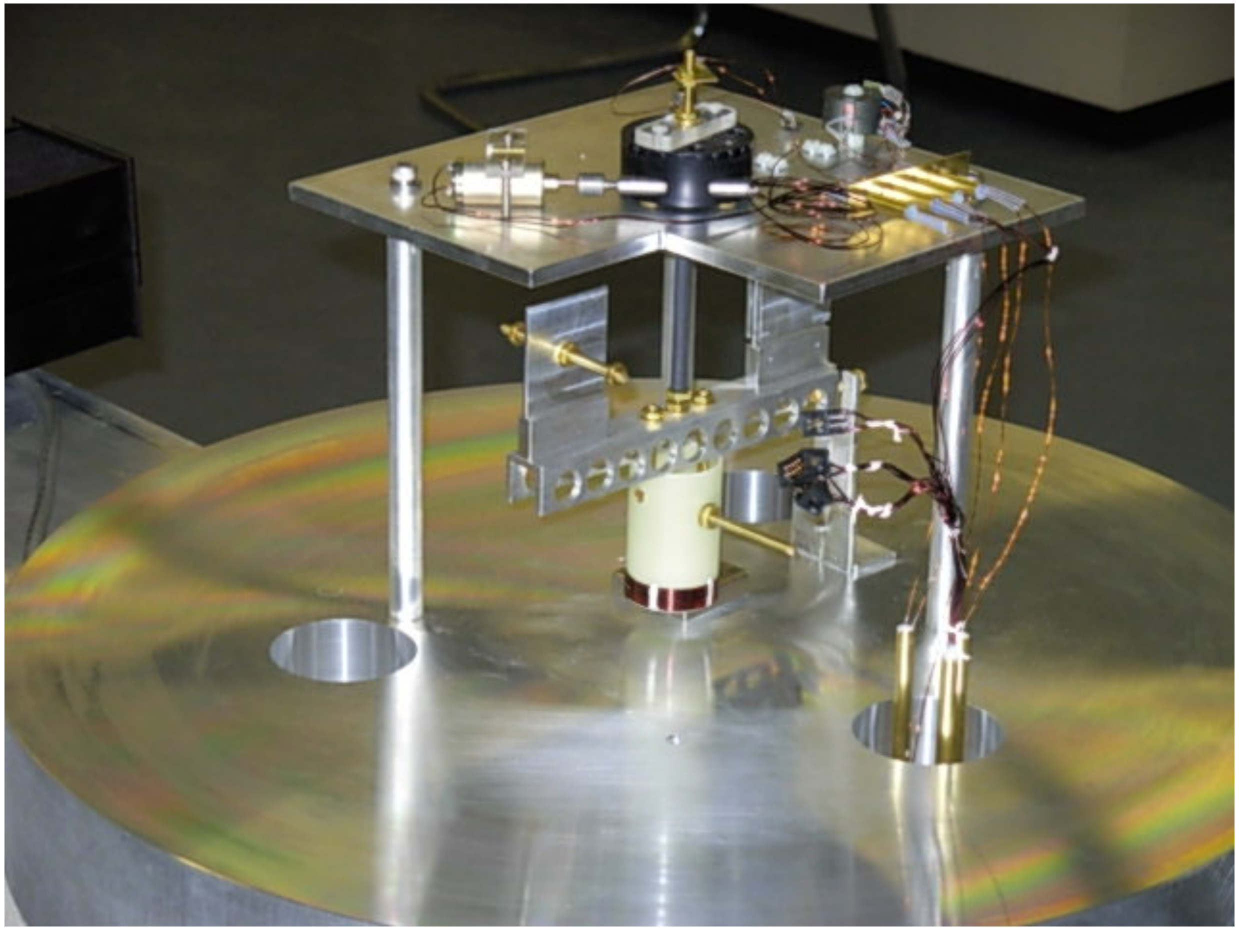
Some limiting cases:

$F \propto d^{-3}$ (non-retarded limit, small distances)

$F \propto d^{-4}$ (retarded limit, larger distances)

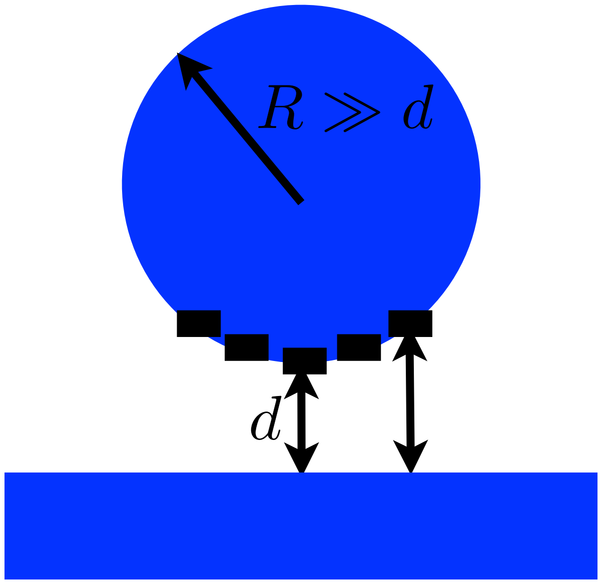
$F \propto Td^{-3}$ (classical limit, very large distances)

How are these forces measured?



Torsional pendulum

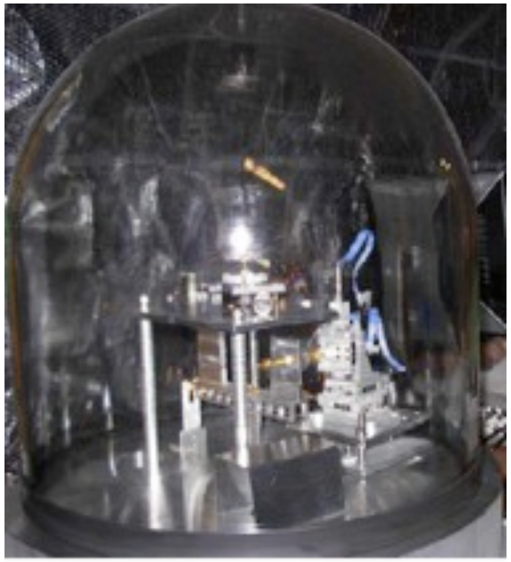
Experiment by Lamoreaux group (Yale)

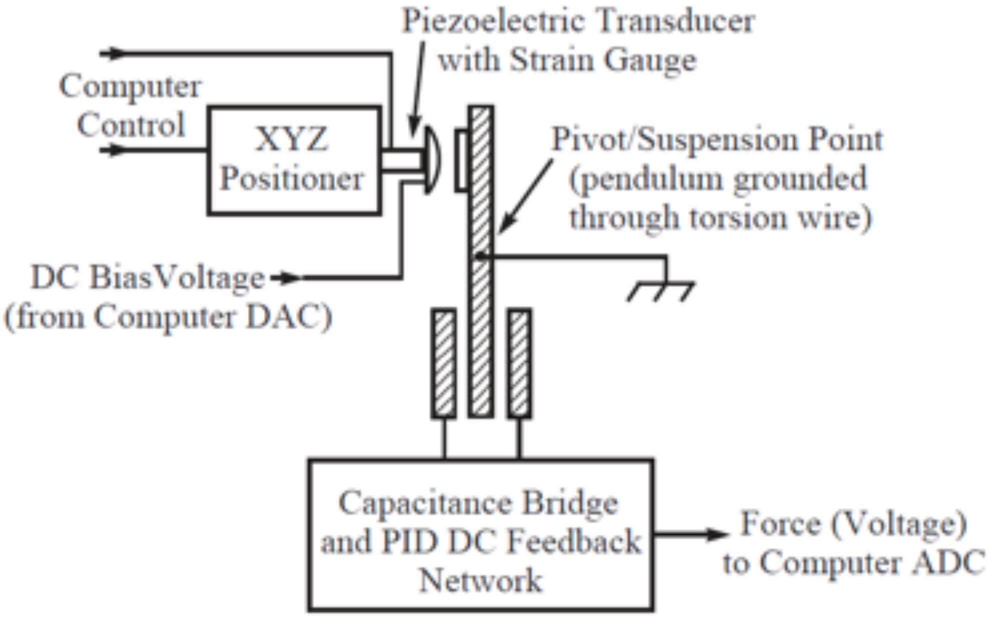


- Sphere-plane geometry: $R = 15.1$ cm

  $d \approx 1\ \mu\mathrm{m}$

- Torsional pendulum (modern Cavendish-like)





Typical Casimir measurement

$$S_{\rm PID}(d, V_a) = S_{\rm dc}(d \to \infty) + S_a(d, V_a) + S_r(d)$$

force-free component of signal at large separations

electrostatic signal in response to an applied external voltage

residual signal due to distance-dependent forces, e.g. Casimir

The electrostatic signal between the spherical lens and the plate, in PFA $(d \ll R)$ is

$$S_a(d, V_a) = \pi\epsilon_0 R (V_a - V_m)^2 / \beta d$$

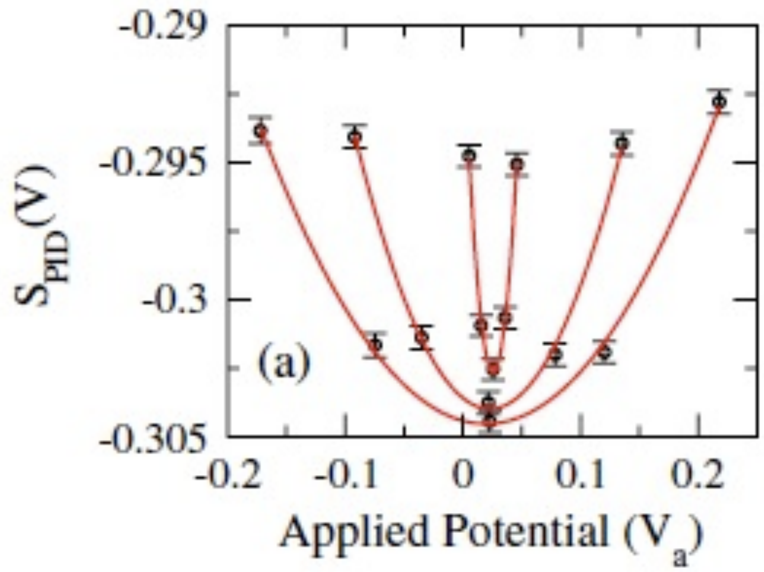
β force-voltage conversion factor

This signal is minimized $(S_a = 0)$ when $V_a = V_m$, and the electrostatic minimizing potential V_m is then defined to be the contact potential between the plates.

“Parabola” measurements

Calibration routine

A range of plate voltages V_a is applied, and at a given nominal absolute distance the response is fitted to a parabola



$$S_{\mathrm{PID}}(d, V_a) = S_0 + k(V_a - V_m)^2$$

Fitting parameters

$k = k(d)$ $\longrightarrow$ voltage-force calibration factor + absolute distance

$V_m = V_m(d)$ $\longrightarrow$ distance-dependent minimizing potential

$S_0 = S_0(d)$ $\longrightarrow$ force residuals: patch potentials + Casimir + non-Newtonian gravity + ...

Metals are not equipotentials

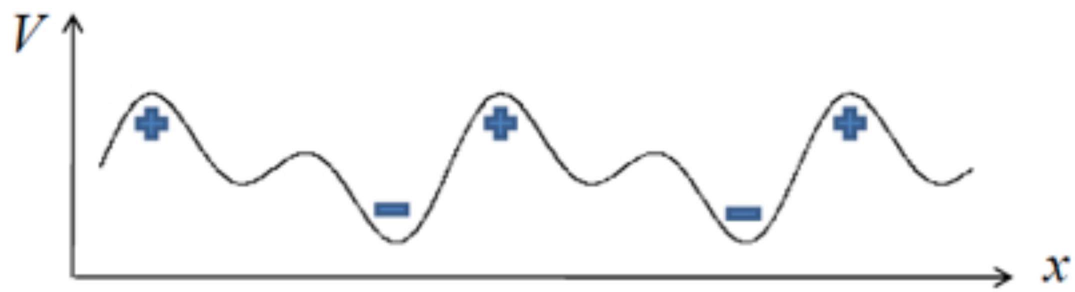
- Despite what we have learned in freshman physics!

- Different crystal faces have different work functions

Au crystal direction	Work function
$\langle 100 \rangle$	5.47 eV
$\langle 110 \rangle$	5.37 eV
$\langle 111 \rangle$	5.31 eV

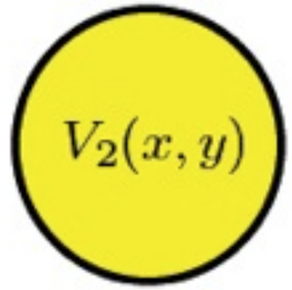
- Dirt: oxides, surface adsorbates strongly affect work function and surface potential by creating dipoles on the surface.

Resulting potential variation across a surface:



Modeling patch potentials

To compute the patch effect in the sphere-plane configuration we use PFA for the curvature effect $(d \ll R)$ but leave kd arbitrary



$$\nabla^2 V(x, y, z) = 0$$

$$\boxed{V(z=0) = V_1(x, y)}$$

$$F_{sp}(d) = 2\pi R \langle U_{pp}(d) \rangle = \frac{\epsilon_0 R}{16} \int_0^\infty dk \, \frac{k^2 e^{-kd}}{\sinh(kd)} \left[C_{1,k} + C_{2,k} \right]$$

Statistical properties for patch potentials:

$$\begin{aligned} \langle V_{1,\mathbf{k}} \rangle &= \langle V_{2,\mathbf{k}} \rangle = \langle V_{2,\mathbf{k}} V_{1,\mathbf{k}'} \rangle = 0; \\ \langle V_{1,\mathbf{k}} V_{1,\mathbf{k}'} \rangle &= C_{1,\mathbf{k}} \, \delta^2(\mathbf{k} - \mathbf{k}'); \\ \langle V_{2,\mathbf{k}} V_{2,\mathbf{k}'} \rangle &= C_{2,\mathbf{k}} \, \delta^2(\mathbf{k} - \mathbf{k}'), \end{aligned}$$

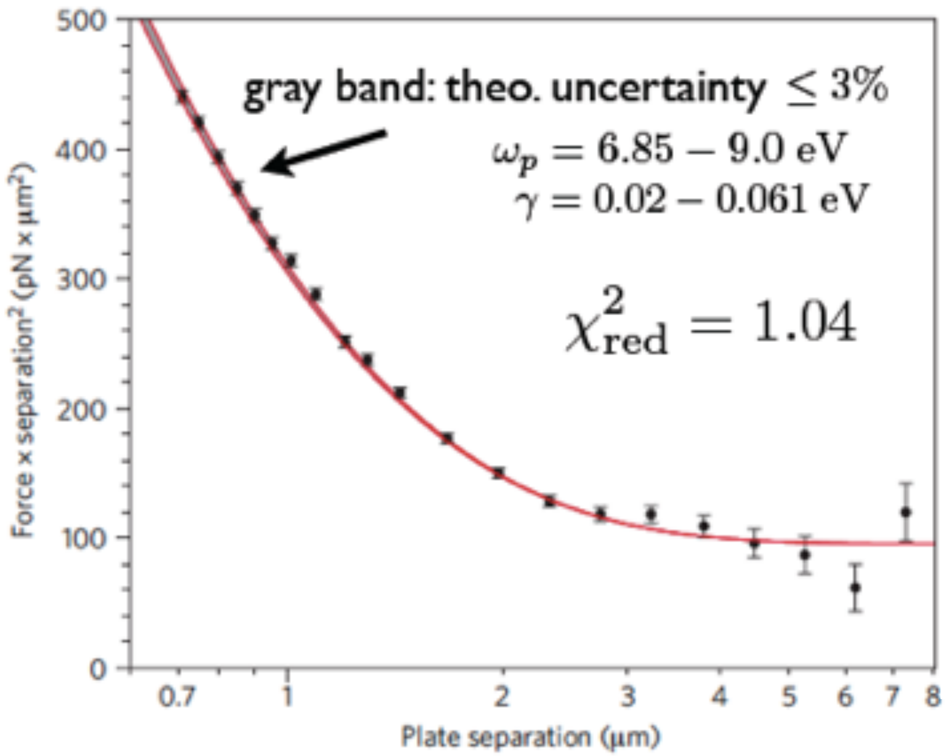
In the limit of large patches $(kd \ll 1)$:

$$\boxed{F_{sp}(d) = \pi \epsilon_0 R \, \frac{V_{\mathrm{rms}}^2}{d}}$$

Speake and Trenkel, PRL 2003

Behunin, DD, Zeng, Reynaud, PRA 2012

Thermal Casimir force



Thermal Casimir force

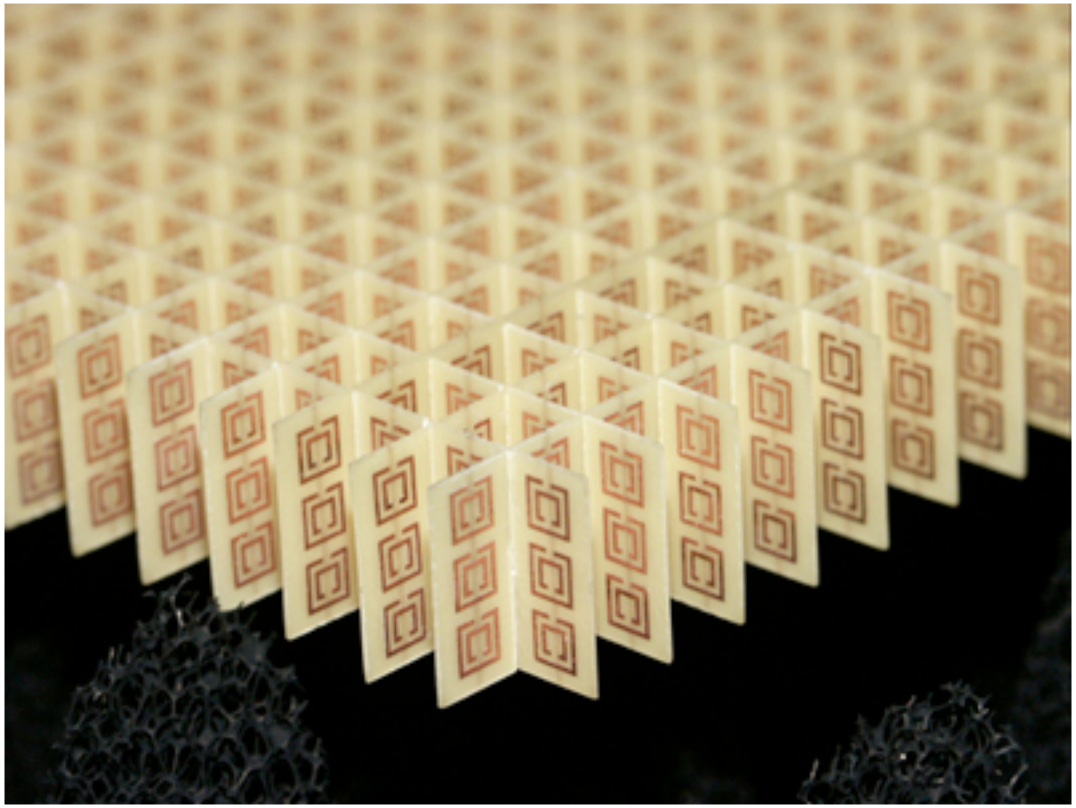
$$F^{(T)}_{\rm Drude} = \frac{\xi(3) R k_{\rm B} T}{8 d^2} = 97\,{\rm pN}\,\mu{\rm m}^2$$

ARTICLES
PUBLISHED ONLINE: 6 FEBRUARY 2011 | DOI: 10.1038/NPHYS1909
nature physics

Observation of the thermal Casimir force

A. O. Sushkov[1]*, W. J. Kim[2], D. A. R. Dalvit[3] and S. K. Lamoreaux[1]

Tailoring Casimir with nanostructures



The sign of the Casimir force

$$\frac{F}{A} = -2k_B T \sum_p \sum_{m=0}^{\infty}{}' \int \frac{d^2\mathbf{k}}{(2\pi)^2} \sqrt{\xi_m^2/c^2 + k^2}\, \frac{R_{1,p}(i\xi_m, k) R_{2,p}(i\xi_m, k) e^{-2d\sqrt{\xi_m/c^2+k^2}}}{1 - R_{1,p}(i\xi_m, k) R_{2,p}(i\xi_m, k) e^{-2d\sqrt{\xi_m/c^2+k^2}}}$$

The sign of the force is directly connected to the sign of the product of the reflection coefficients on the two plates, evaluated at imaginary frequencies. As a rule of thumb, we have (p=TE, TM)

$$R_{1,p}(i\xi_m, k) R_{2,p}(i\xi_m, k) > 0 \;\Rightarrow\; \text{Attraction}$$

$$R_{1,p}(i\xi_m, k) R_{2,p}(i\xi_m, k) < 0 \;\Rightarrow\; \text{Repulsion}$$

In terms of permittivities and permeabilities:

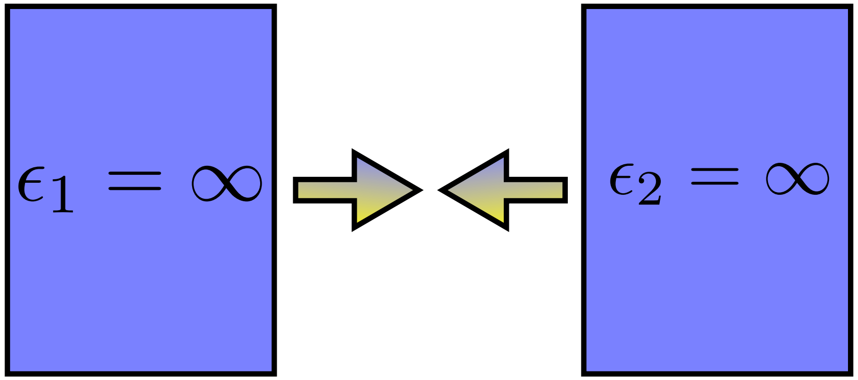
$$\left.\begin{array}{l} \epsilon_a(i\xi) \gg \epsilon_b(i\xi) \\ \mu_b(i\xi) \gg \mu_a(i\xi) \end{array}\right. \longrightarrow \text{Repulsion}$$

Ideal attraction-repulsion

- **Ideal attractive limit**

  Casimir (1948)

$$\boxed{\frac{F}{A} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$

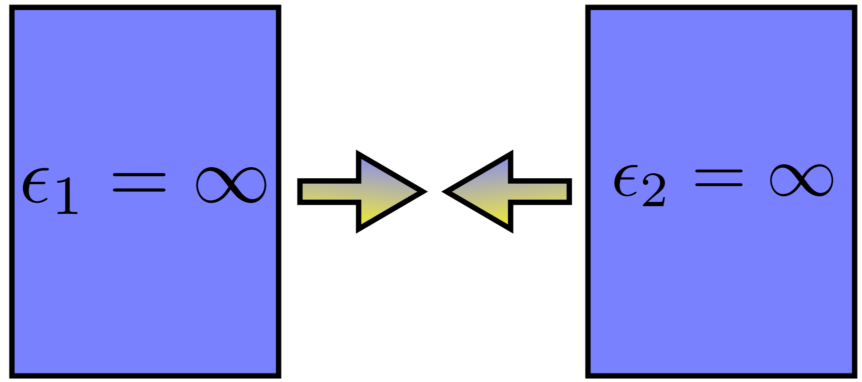


Ideal attraction-repulsion

- **Ideal attractive limit**

  Casimir (1948)

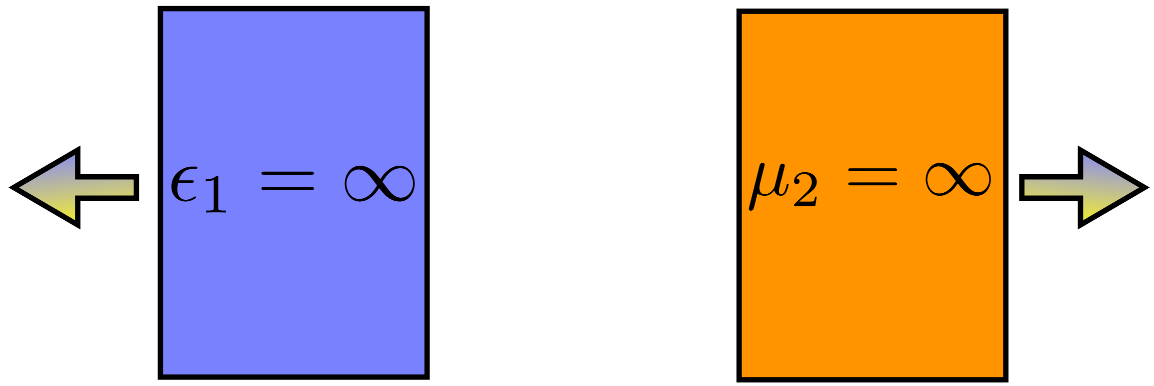
$$\boxed{\frac{F}{A} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



- **Ideal repulsive limit**

  Boyer (1974)

$$\boxed{\frac{F}{A} = \frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$

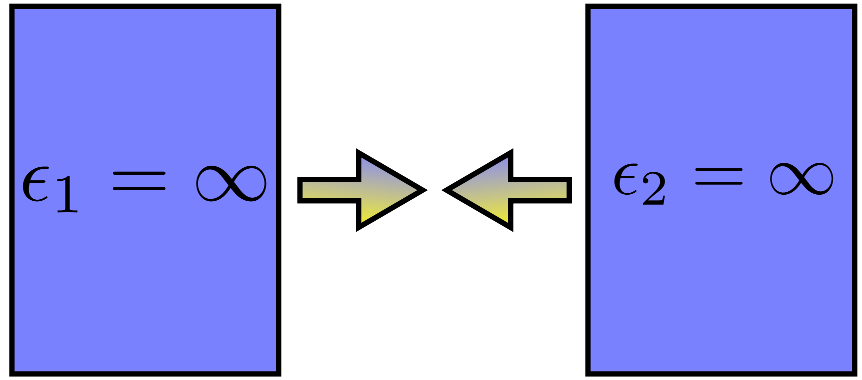


Ideal attraction-repulsion

- ## Ideal attractive limit

Casimir (1948)

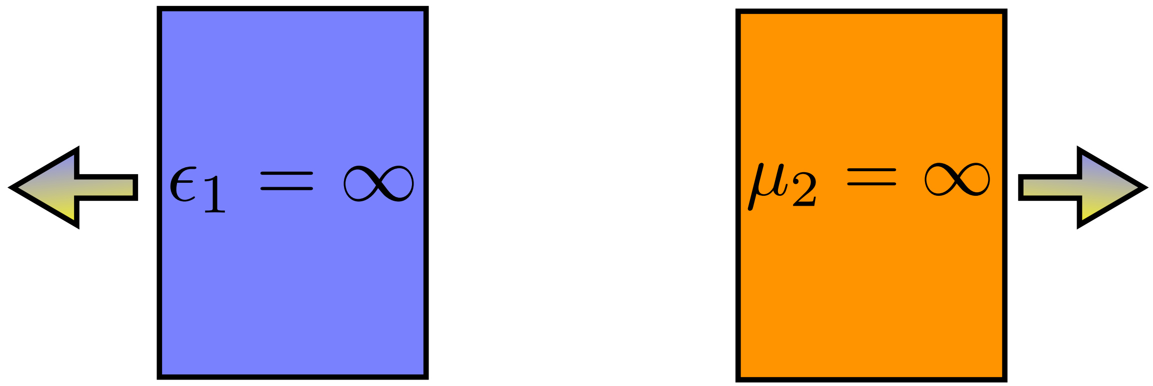
$$\boxed{\frac{F}{A} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



- ## Ideal repulsive limit

Boyer (1974)

$$\boxed{\frac{F}{A} = \frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



- ## Real repulsive limit

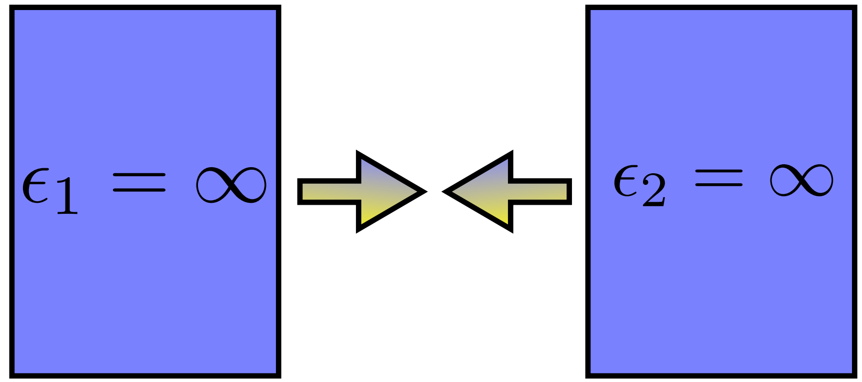
Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$

Ideal attraction-repulsion

- **Ideal attractive limit**

  Casimir (1948)

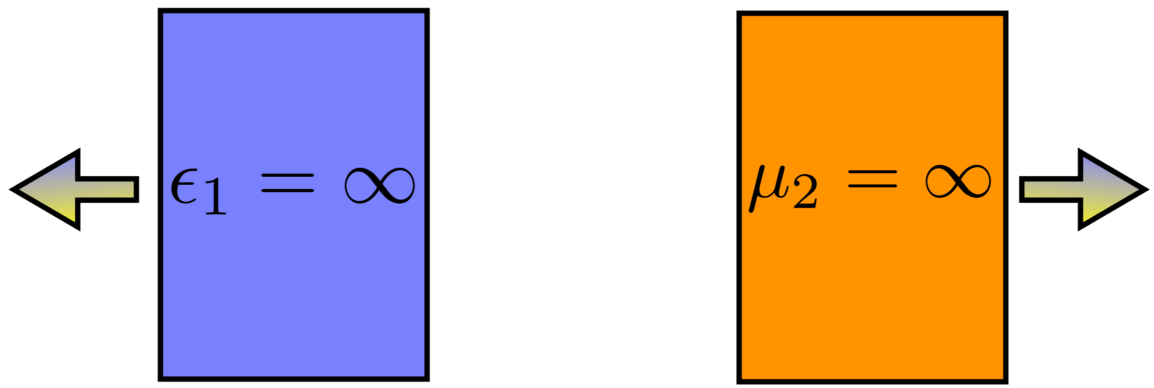
$$\boxed{\frac{F}{A} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



- **Ideal repulsive limit**

  Boyer (1974)

$$\boxed{\frac{F}{A} = \frac{7}{8}\frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$



- **Real repulsive limit**

  Casimir repulsion is associated with strong electric-magnetic interactions. However, natural occurring materials do NOT have strong magnetic response in the optical region, i.e. $\mu = 1$ $\longrightarrow$ Metamaterials

Quantum levitation with MMs?

Physicists have 'solved' mystery of levitation

By Roger Highfield, Science Editor
Last Updated: 1:41am BST 08/08/2007

Levitation has been elevated from being pure science fiction to science fact, according to a study reported today by physicists.

In earlier work the same team of theoretical physicists showed that invisibility cloaks are feasible.

Now, in another report that sounds like it comes out of the pages of a Harry Potter book, the University of St Andrews team has created an 'incredible levitation effects' by engineering the force of nature which normally causes objects to stick together.

Professor Ulf Leonhardt and Dr Thomas Philbin, from the University of St Andrews in Scotland, have worked out a way of reversing this phenomenon, known as the Casimir force, so that it repels instead of attracts.

Their discovery could ultimately lead to frictionless micro-machines with moving parts that levitate But they say that, in principle at least, the same effect could be used to levitate bigger objects too, even a person.

In theory the discovery could be used to levitate a person

The Casimir force is a consequence of quantum mechanics, the theory that describes the world of atoms and subatomic particles that is not only the most successful theory of physics but also the most baffling.

The force is due to neither electrical charge or gravity, for example, but the fluctuations in all-pervasive energy fields in the intervening empty space between the objects and is one reason atoms stick together, also explaining a "dry glue" effect that enables a gecko to walk across a ceiling.

Now, using a special lens of a kind that has already been built, Prof Ulf Leonhardt and Dr Thomas Philbin report in the New Journal of Physics they can engineer the Casimir force to repel, rather than attact.

Because the Casimir force causes problems for nanotechnologists, who are trying to build electrical circuits and tiny mechanical devices on silicon chips, among other things, the team believes the feat could initially be used to stop tiny objects from sticking to each other.

Prof Leonhardt explained, "The Casimir force is the ultimate cause of friction in the nano-world, in particular in some microelectromechanical systems.

Such systems already play an important role - for example tiny mechanical devices which triggers a car airbag to inflate or those which power tiny 'lab on chip' devices used for drugs testing or chemical analysis.

Micro or nano machines could run smoother and with less or no friction at all if one can manipulate the force." Though it is possible to levitate objects as big as humans, scientists are a long way off developing the technology for such feats, said Dr Philbin.

The practicalities of designing the lens to do this are daunting but not impossible and levitation "could happen over quite a distance".

Prof Leonhardt leads one of four teams - three of them in Britain - to have put forward a theory in a peer-reviewed journal to achieve invisibility by making light waves flow around an object - just as a river flows undisturbed around a smooth rock.

Print | Email this story

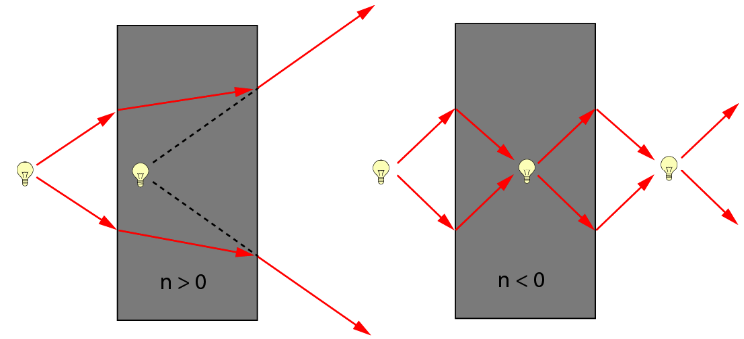
Post this story to: del.icio.us | Digg | Newsvine | NowPublic | Reddit

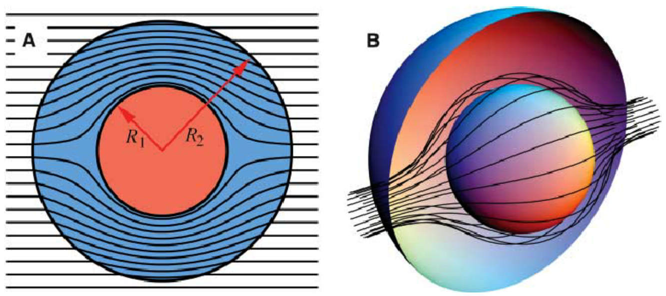
"In theory the discovery could be used to levitate a person"

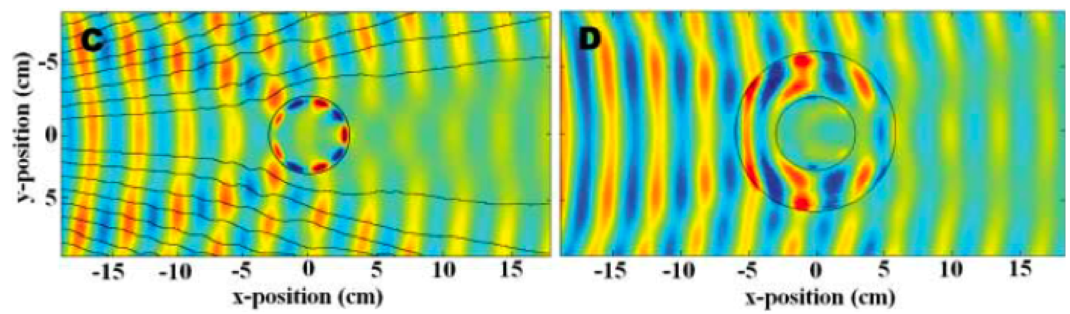
Metamaterials

- Artificial structured composites with designer electromagnetic properties
- MMs are strongly anisotropic, dispersive, magneto-dielectric media.

- Negative refraction — Veselago (1968), Smith et al (2000)
- Perfect lens — Pendry (2000)
- Cloaking — Smith et al (2007)

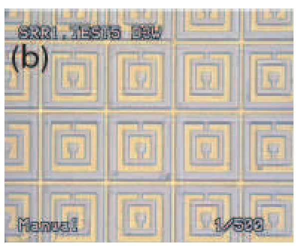


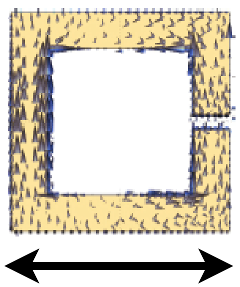




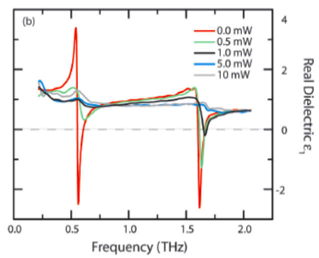
THz MMs: eg split ring resonators

Optical MMs: eg nano-pillars

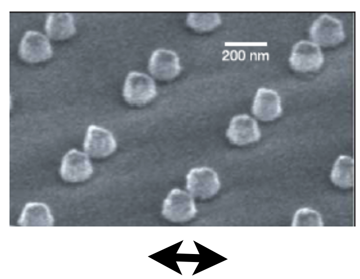




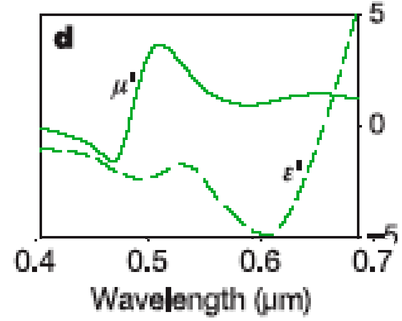
36μm



$\epsilon, \mu < 0$

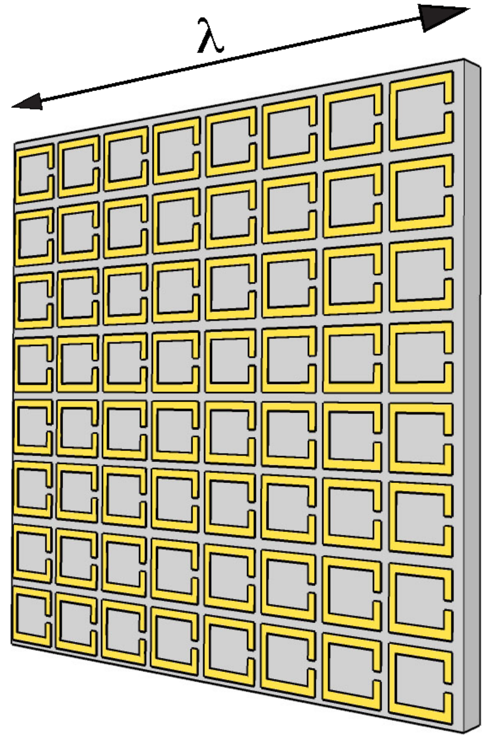


200nm



Effective medium approximation

Imagine that the MM is probed at wavelengths much larger that the average distance between the constituent "particles" of the MM.



In this situation the MM is effectively a continuous medium, whose optical response can be characterized by an effective electric permittivity and an effective magnetic permeability.

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\omega\Gamma}$$

$$\mu_{eff} = 1 - \frac{\frac{\pi r^2}{a^2}}{1 + \frac{2\sigma i}{\omega r \mu_0} - \frac{3}{\pi^2 \mu_0 \omega^2 C r^3}}$$

EMA: Drude-Lorentz responses

Close to the resonance, both $\epsilon(\omega)$ and $\mu(\omega)$ can be modeled by Drude-Lorentz formulas

$$\boxed{\begin{aligned}\epsilon_\alpha(\omega) &= 1 - \frac{\Omega^2_{E,\alpha}}{\omega^2 - \omega^2_{E,\alpha} + i\Gamma_{E,\alpha}\omega} \\ \mu_\alpha(\omega) &= 1 - \frac{\Omega^2_{M,\alpha}}{\omega^2 - \omega^2_{M,\alpha} + i\Gamma_{M,\alpha}\omega}\end{aligned}}$$

Typical separations

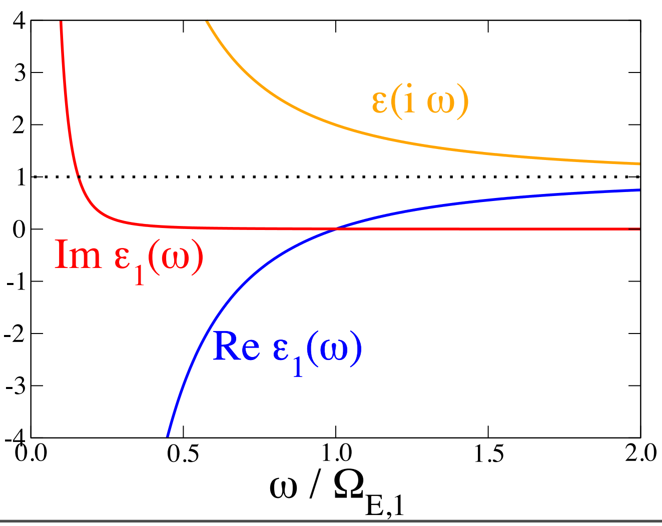
$$d = 200 - 1000 \text{ nm}$$

Infrared-optical frequencies

$$\Omega/2\pi = 5 \times 10^{14} \text{Hz}$$

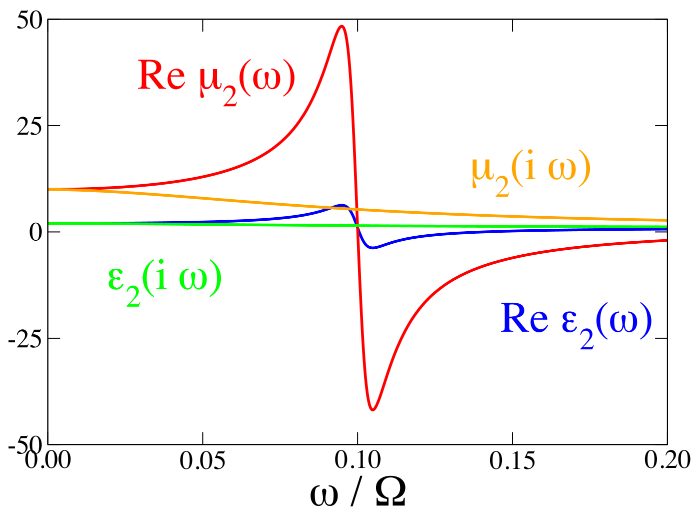
Drude metal (Au)

$\Omega_E = 9.0$ eV $\quad \Gamma_E = 35$ meV



Metamaterial

Re $\epsilon_2(\omega) < 0 \quad$ Re $\mu_2(\omega) < 0$



$$\Omega_{E,2}/\Omega = 0.1 \quad \Omega_{M,2}/\Omega = 0.3$$

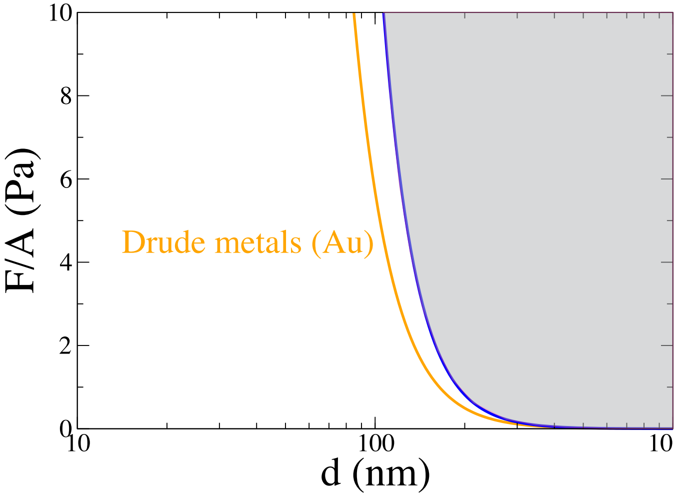
$$\omega_{E,2}/\Omega = \omega_{M,2}/\Omega = 0.1$$

$$\Gamma_{E,2}/\Omega = \Gamma_{M,2}/\Omega = 0.01$$

Attraction-repulsion crossover

Drude metal (Au)

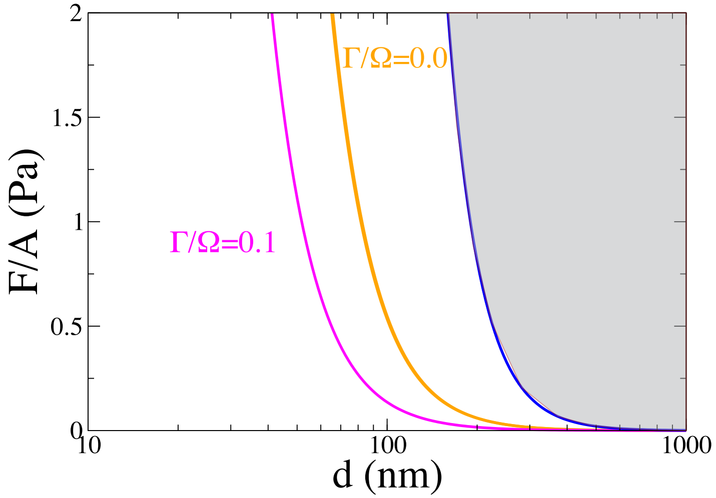
Drude metal (Au)



Only attraction

Metamaterial

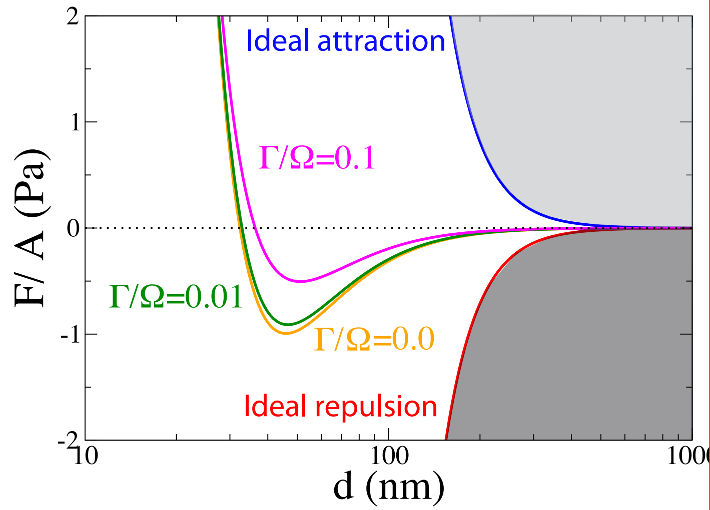
Metamaterial



Only attraction

Drude metal (Au)

Metamaterial



Repulsion-attraction

EMA: correct model for μ

Drude-Lorentz for permeability is wrong. The correct expression that results in EMA from Maxwell's equations is

$$\boxed{\mu_{\text{eff}}(\omega) = 1 - f\frac{\omega^2}{\omega^2 - \omega_m^2 + 2i\gamma_m\omega}}$$

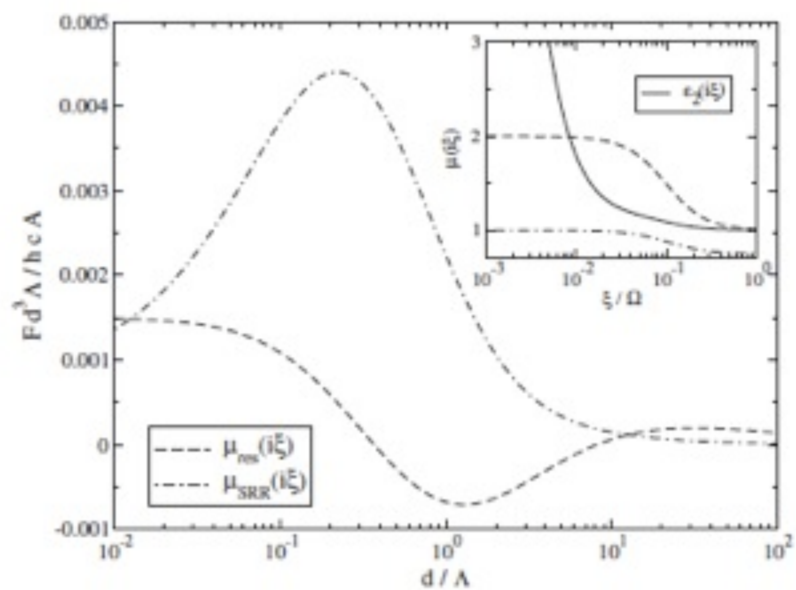
(Pendry 1999)

The appearance of the ω^2 factor in the numerator is very important:

Although close to the resonance this behaves in the same way as the Drude-Lorentz EMA permeability, it has a completely different low-frequency behavior

$$\mu_{\text{eff}}(i\xi) < 1 < \epsilon_{\text{eff}}(i\xi)$$

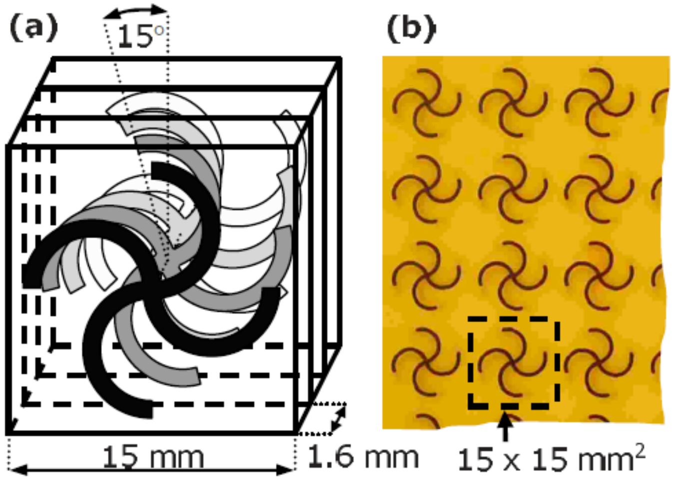
No Casimir repulsion!

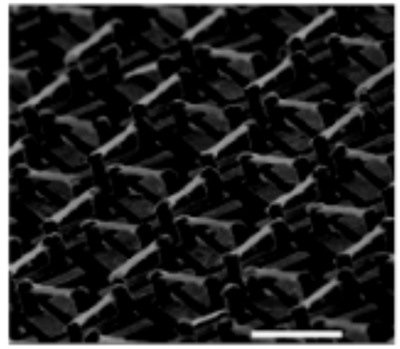


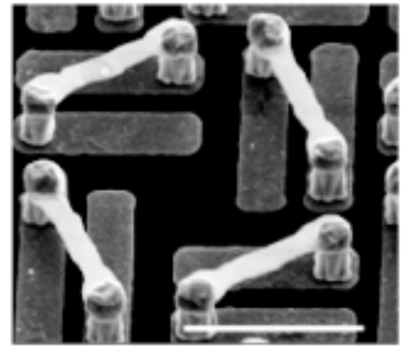
Rosa, DD, Milonni, PRL **100**, 183602 (2008)

Other Casimir MMs: chirality

The chirality of a MM is defined by the chirality of its unit cell







In a chiral medium, the constitutive relations mix electric and magnetic fields

$$\mathbf{D}(\mathbf{r},\omega) = \epsilon(\omega)\mathbf{E}(\mathbf{r},\omega) - i\kappa(\omega)\mathbf{H}(\mathbf{r},\omega)$$
$$\mathbf{B}(\mathbf{r},\omega) = i\kappa(\omega)\mathbf{E}(\mathbf{r},\omega) + \mu(\omega)\mathbf{H}(\mathbf{r},\omega)$$

dispersive chirality: $\kappa(\omega) = \dfrac{\omega_k\omega}{\omega^2 - \omega_{\kappa R}^2 + i\gamma_k\omega}$

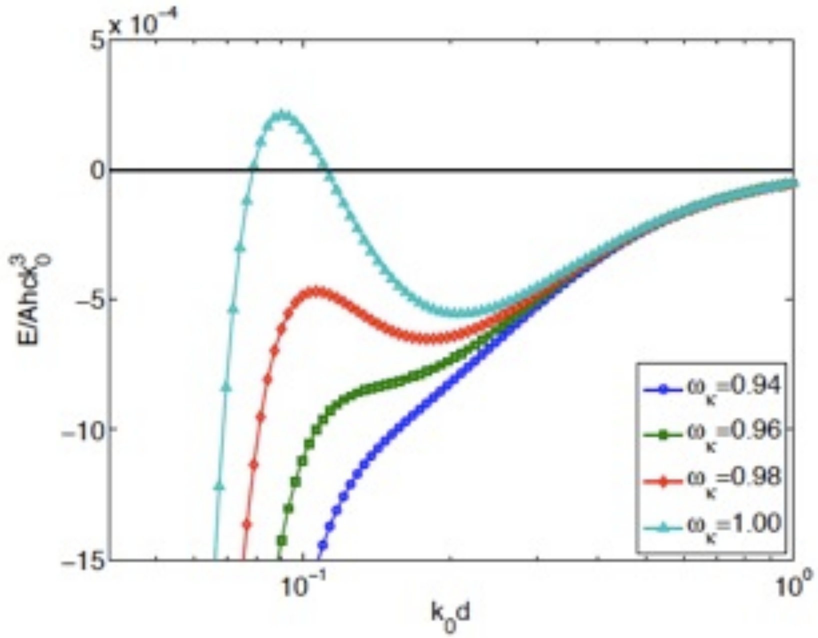
Repulsion and chiral MMs

In chiral MMs the reflection matrix is non-diagonal (mixing of E and H fields).

The integrand of the Casimir-Lifshitz force between two identical chiral MMs has the form:

$$F = \frac{(r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} - 2(r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}{1 - (r_{ss}^2 + r_{pp}^2 - 2r_{sp}^2)e^{-2Kd} + (r_{sp}^2 + r_{ss}r_{pp})^2 e^{-4Kd}}$$

One might achieve repulsive Casimir forces with strong chirality (i.e., large values of r_{sp})



Same-chirality materials: repulsion

Opposite-chirality materials: repulsion

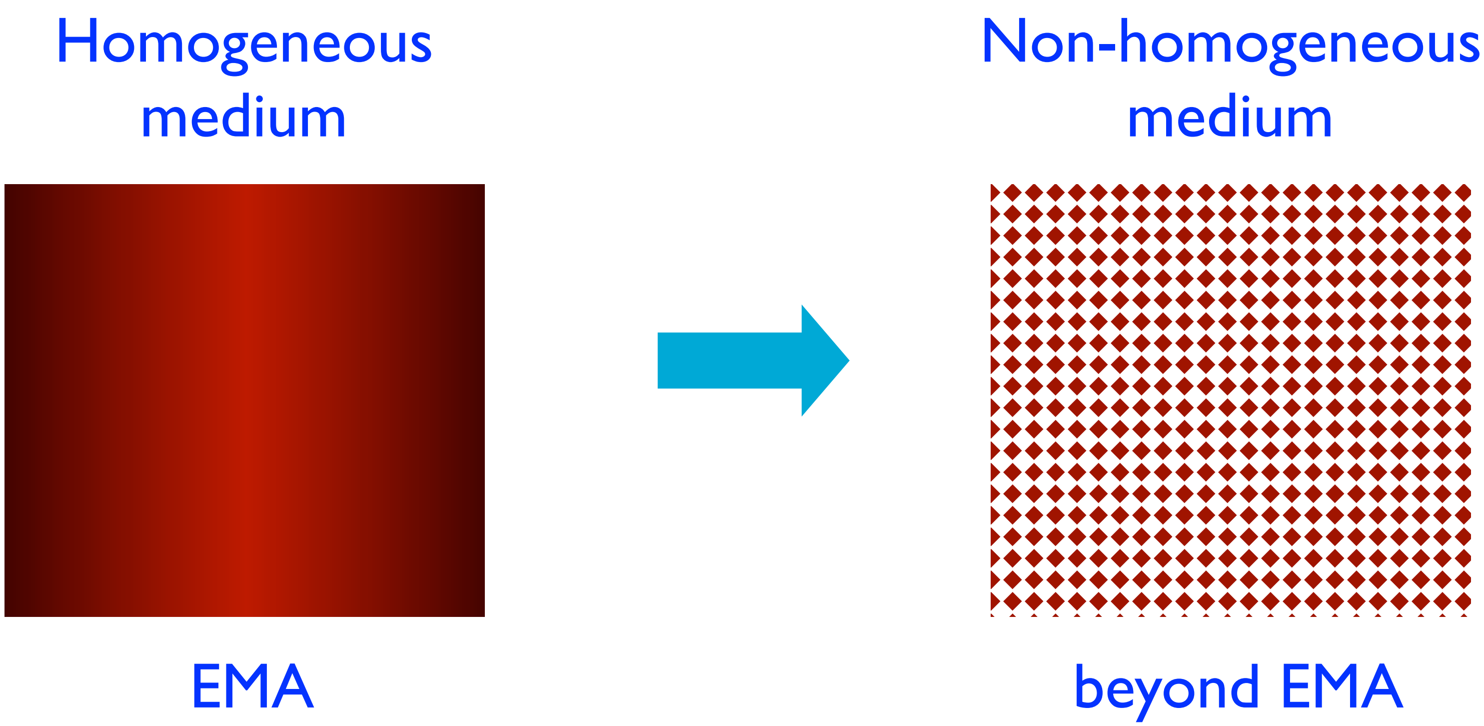
However, EMA breaks down here!

Soukoulis *et al.*, PRL 2009

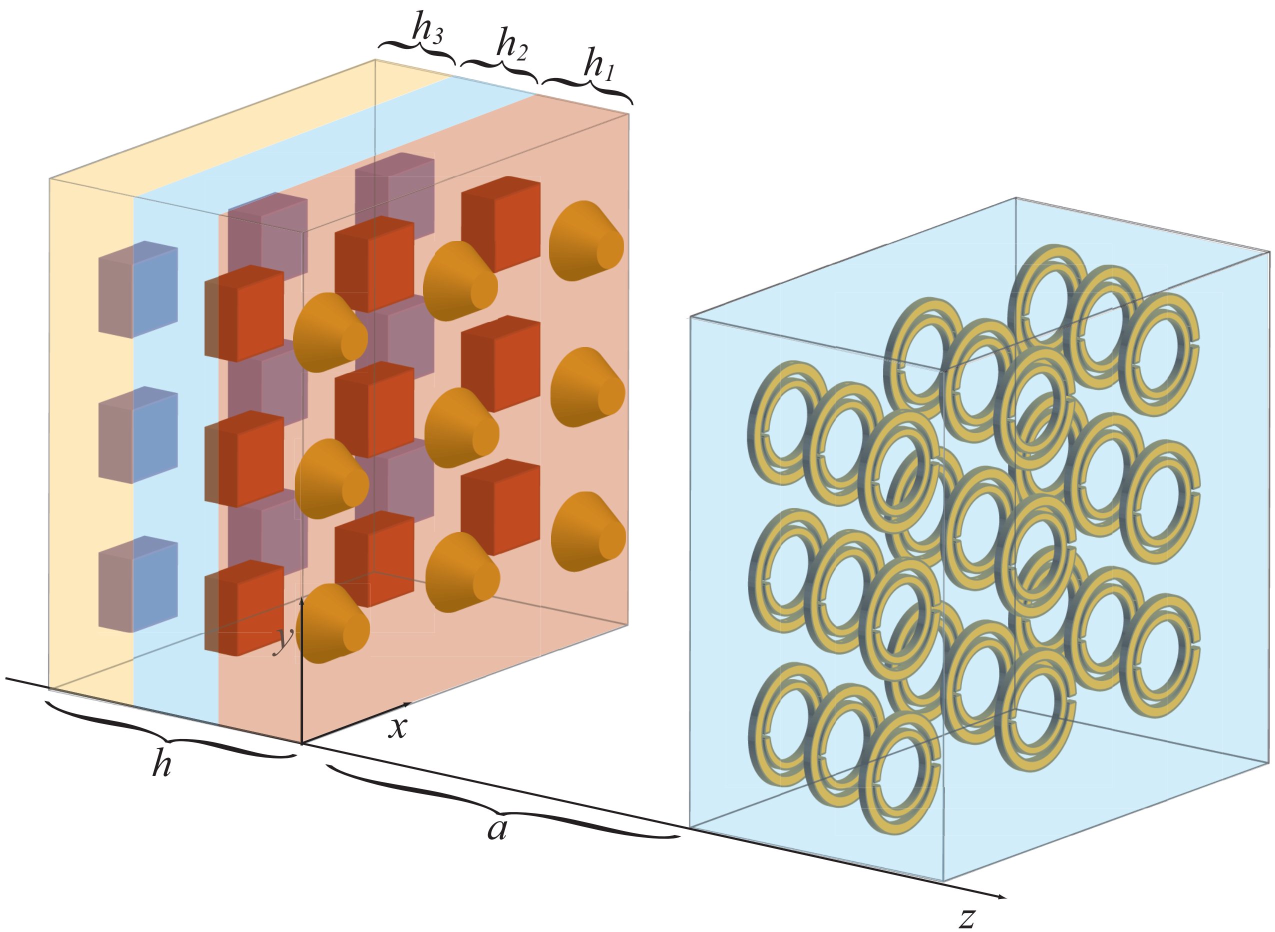
Going beyond EMA

So far, we have treated the MM in the "long-wavelength approximation", i.e., field wavelengths much larger than the typical size of the unit cell of the MM.

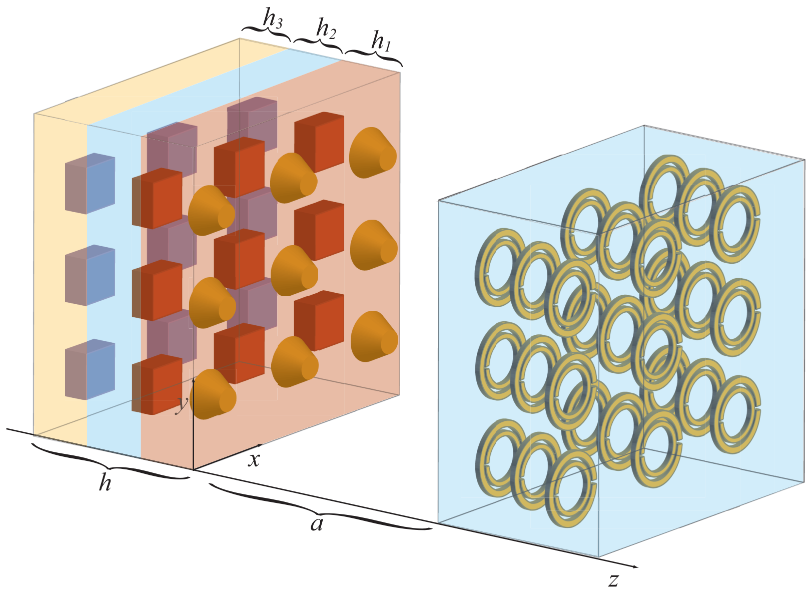
How to calculate Casimir forces when EMA does not hold?
Can one trust predictions of Casimir repulsion with MMs based on EMA?



Casimir nanostructures

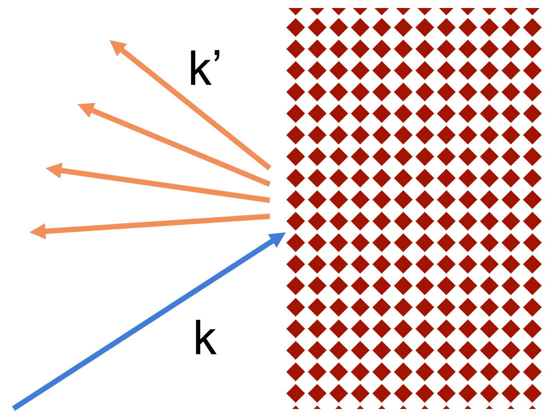


Scattering theory



The Casimir force still may be described in terms of reflections (scattering theory)

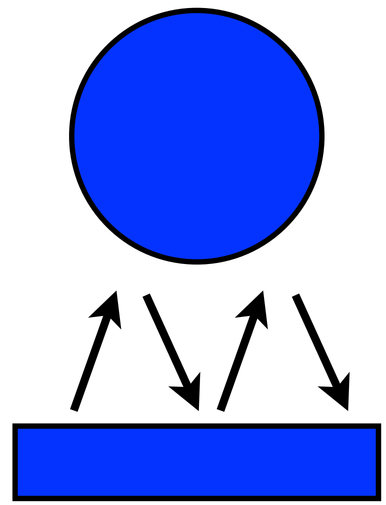
$$\mathcal{R}_i(\omega, \mathbf{k}, \mathbf{k}', p, p')$$



Symbolically, we may write the Casimir energy as

$$\boxed{\frac{E(d)}{A} = \hbar \int_0^\infty \frac{d\xi}{2\pi} \log\det\left[1 - \mathcal{R}_1 e^{-\mathcal{K}d} \mathcal{R}_2 e^{-\mathcal{K}d}\right]}$$

$$\propto \sum_{n=1}^{\infty} \frac{1}{n} \left[\mathcal{R}_1(i\xi) e^{-d\mathcal{K}(i\xi)} \mathcal{R}_2(i\xi) e^{-d\mathcal{K}(i\xi)}\right]^n$$



Solving for the reflection matrix

The reflection matrix can be obtained with standard methods of numerical electromagnetism. One way is to solve Maxwell equations for the transverse fields

$$-ik\frac{\partial \mathbf{E}_t}{\partial z} = \nabla_t\left[\chi\hat{e}_3 \cdot \nabla \times \mathbf{H}_t\right] - k^2\mu\hat{e}_3 \times \mathbf{H}_t$$

$$-ik\frac{\partial \mathbf{H}_t}{\partial z} = -\nabla_t\left[\zeta\hat{e}_3 \cdot \nabla \times \mathbf{E}_t\right] + k^2\epsilon\hat{e}_3 \times \mathbf{E}_t$$

Assuming a two-dimensional periodic structure, we have

$$\mathbf{E}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{m,n} \mathcal{E}_{m,n} \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

$$\mathbf{H}_t(x,y) = e^{i\mathbf{k}\cdot\mathbf{r}} \sum_{m,n} \mathcal{H}_{m,n} \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

where

$$\epsilon(x,y) = \sum_{m,n} \epsilon_{m,n} \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

$$\mu(x,y) = \sum_{m,n} \mu_{m,n} \exp\left[i\frac{2\pi n}{L_x}x + i\frac{2\pi m}{L_y}y\right]$$

Exact reflection matrix

One can then write the equations for the transverse fields as

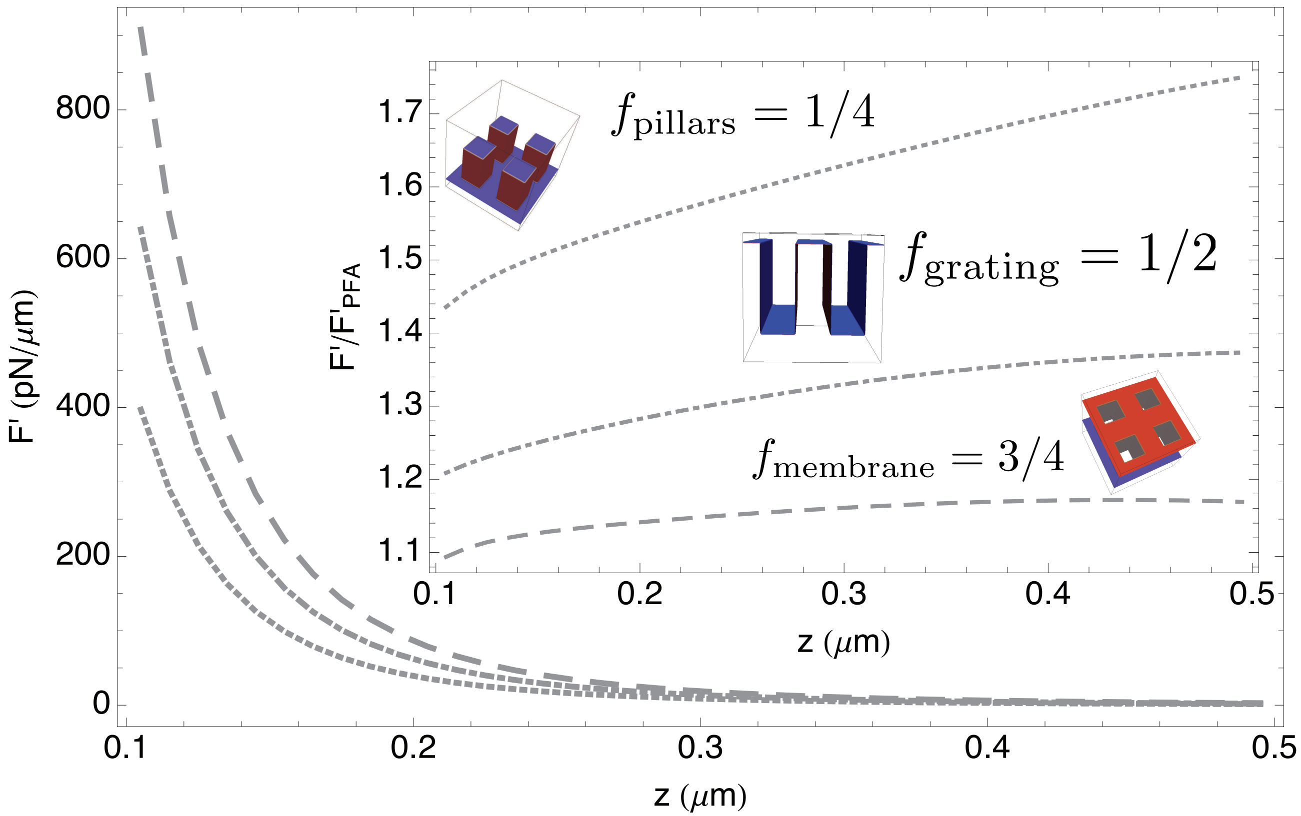
$$\boxed{-ik\frac{\partial \mathbf{\Psi}_{m'n'}}{\partial z} = \sum_{mn} H_{m'n',mn}\mathbf{\Psi}_{mn}} \qquad \mathbf{\Psi}_{mn} = \begin{bmatrix} \mathcal{E}^x_{mn} \\ \mathcal{E}^y_{mn} \\ \mathcal{H}^x_{mn} \\ \mathcal{E}^y_{mn} \end{bmatrix} = \begin{bmatrix} \Psi^1_{mn} \\ \Psi^2_{mn} \\ \Psi^3_{mn} \\ \Psi^4_{mn} \end{bmatrix}$$

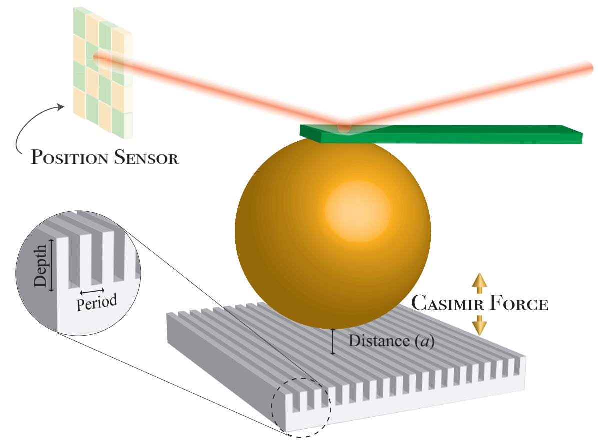
Here H is a complicated matrix, that encapsulates the coupling of modes in the periodic structure.

By numerically solving this equation and imposing the proper boundary conditions of the field on the vacuum-metamaterial interphase (RCWA or S-matrix techniques), one can find the reflection matrix of the MM.

2D periodic structures

Casimir force between a Au plane and Si pillars/grating/membrane @ T=300 K





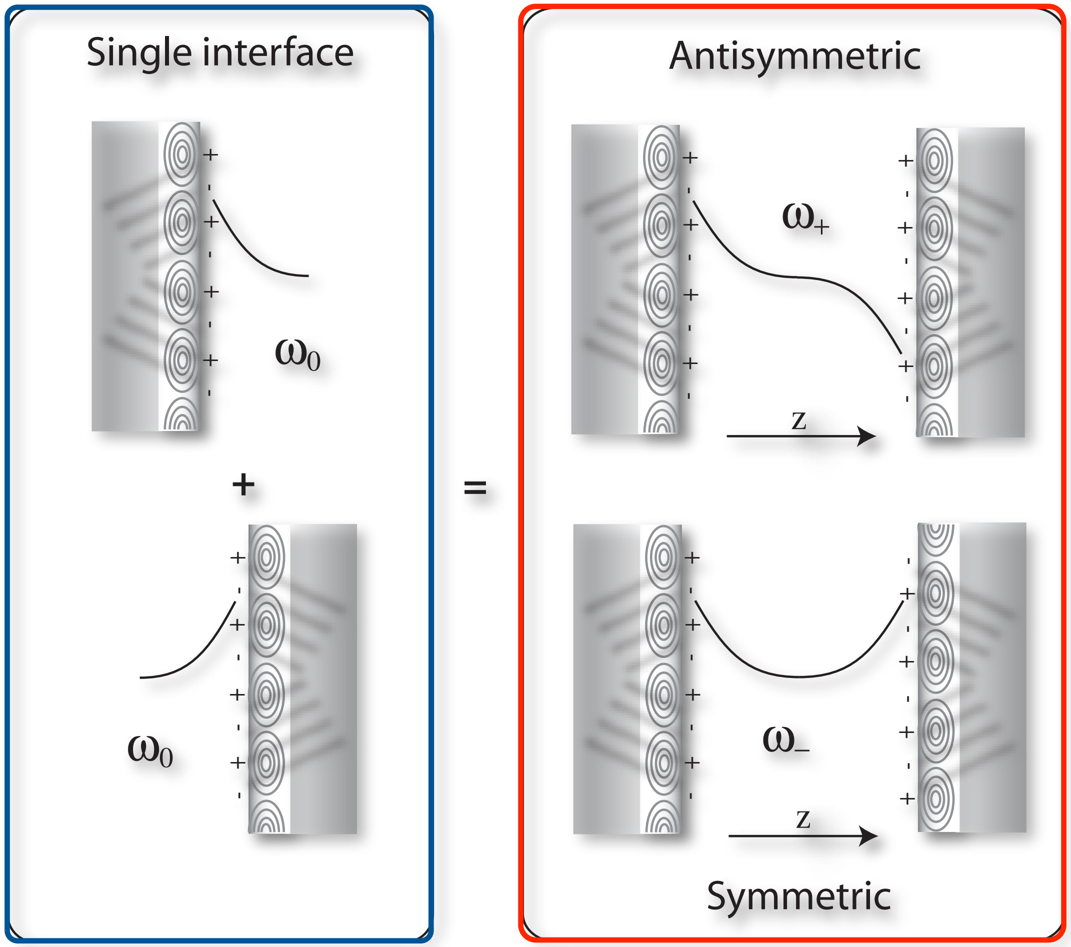
$$R = 50\mu\text{m}$$

$$\text{period} = 400 \text{ nm}$$

$$\text{depth} = 1070 \text{ nm}$$

Davids, Intravaia, Rosa, DD, PRA **82**, 062111 (2010)

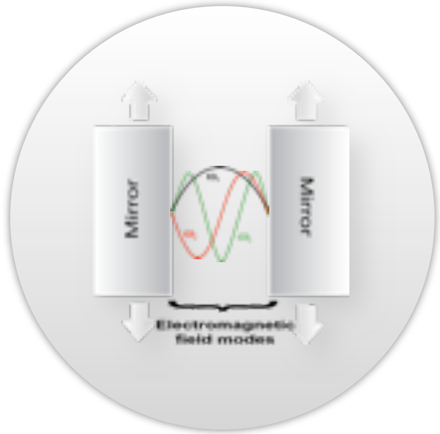
Casimir plasmonics



Mode summation approach

An alternative approach to the scattering formulation is to compute the Casimir energy as a sum over the zero-point energy of the EM in the presence of boundaries

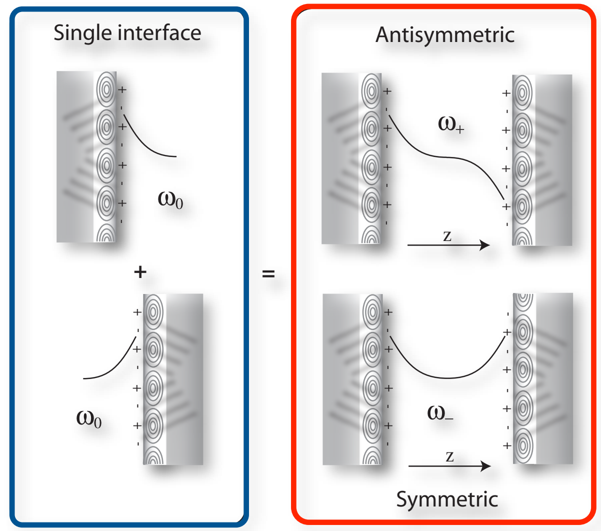
$$E = \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_n \omega_n^p \right]_L}_{\text{Infinite zero point energy}} \underbrace{- \sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_n \omega_n^p \right]_{L\to\infty}}_{\text{Setting the zero}}$$



In the case of metallic plates described by the plasma model

$$\left. \begin{aligned} \mu[\omega] &= 1 \\ \epsilon[\omega] &= 1 - \frac{\omega_p^2}{\omega^2} \end{aligned} \right\} \rightarrow \qquad E = \underbrace{\sum_{\mathbf{k}} \frac{\hbar}{2} \left[\omega_+ + \omega_- \right]_{L\to\infty}^{L}}_{\text{Plasmonic contribution } (E_{pl})} + \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_m \omega_m^p \right]_{L\to\infty}^{L}}_{\text{Photonic contribution } (E_{ph})}$$

Surface plasmons interaction



Surface plasmons are evanescent modes of the EM field associated with electronic density oscillations at the metal-vacuum interface.

When the tails of the evanescent fields overlap, the two surface plasmons hybridize

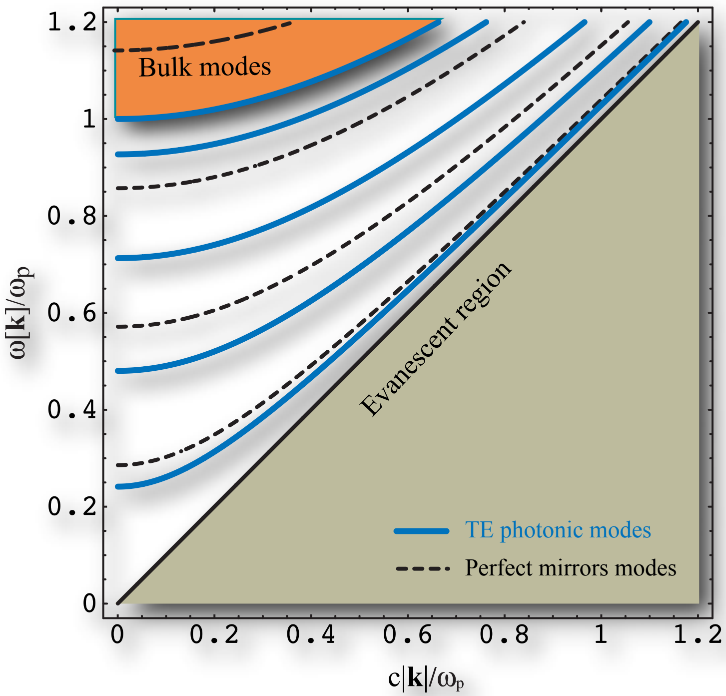
$$2\times\omega_{sp}[\mathbf{k}] \rightarrow \begin{cases}\omega_+[\mathbf{k}] \\ \omega_-[\mathbf{k}]\end{cases}$$

At short distances the Casimir energy is given by the shift in the zero-point energy of the surface plasmons due to their Coulomb (electrostatic) interaction

$$E_{sp} = A\int \frac{d^2\mathbf{k}}{(2\pi)^2}\left(\frac{\hbar\omega_+}{2} + \frac{\hbar\omega_-}{2} - 2\frac{\hbar\omega_{sp}}{2}\right) = -\frac{\hbar c\alpha\pi^2 A}{580\lambda_p L^2}$$

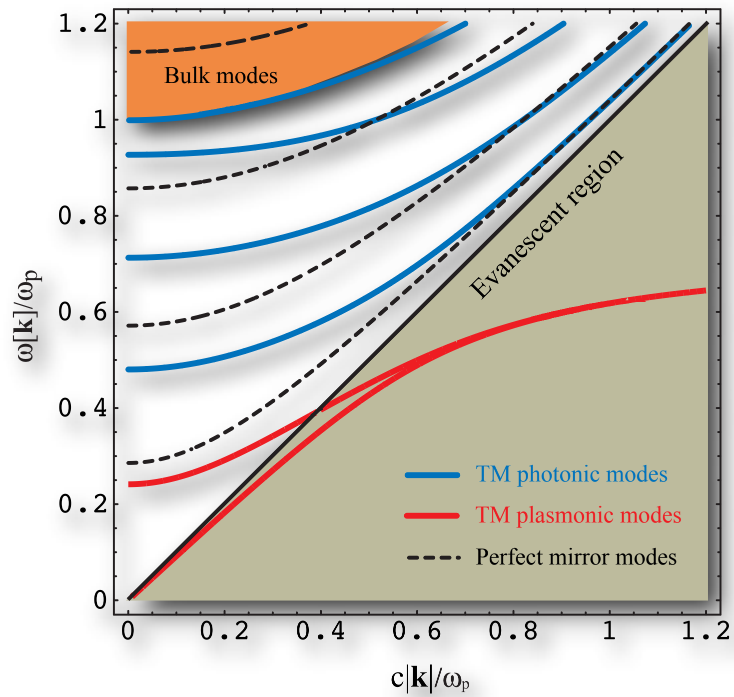
Mode spectrum in a cavity

$$E = \underbrace{\sum_{\mathbf{k}} \frac{\hbar}{2} \left[\omega_+ + \omega_-\right]^L_{L\to\infty}}_{\text{Plasmonic contribution } (E_{pl})} + \underbrace{\sum_{p,\mathbf{k}} \frac{\hbar}{2} \left[\sum_m \omega_m^p\right]^L_{L\to\infty}}_{\text{Photonic contribution } (E_{ph})}$$



All the TE-modes belong to the propagative sector

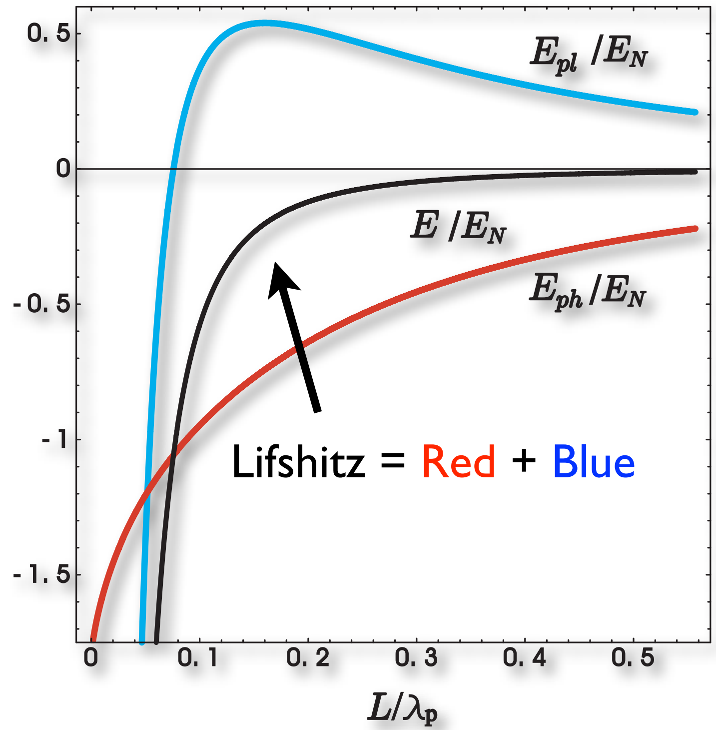
They differ from the perfect mirrors modes because of the dephasing due to the non perfect reflection coefficient.



TM-modes propagative modes look qualitatively like TE modes.

There are only two evanescent modes. They are the generalization to all distances of the coupled plasmon modes.

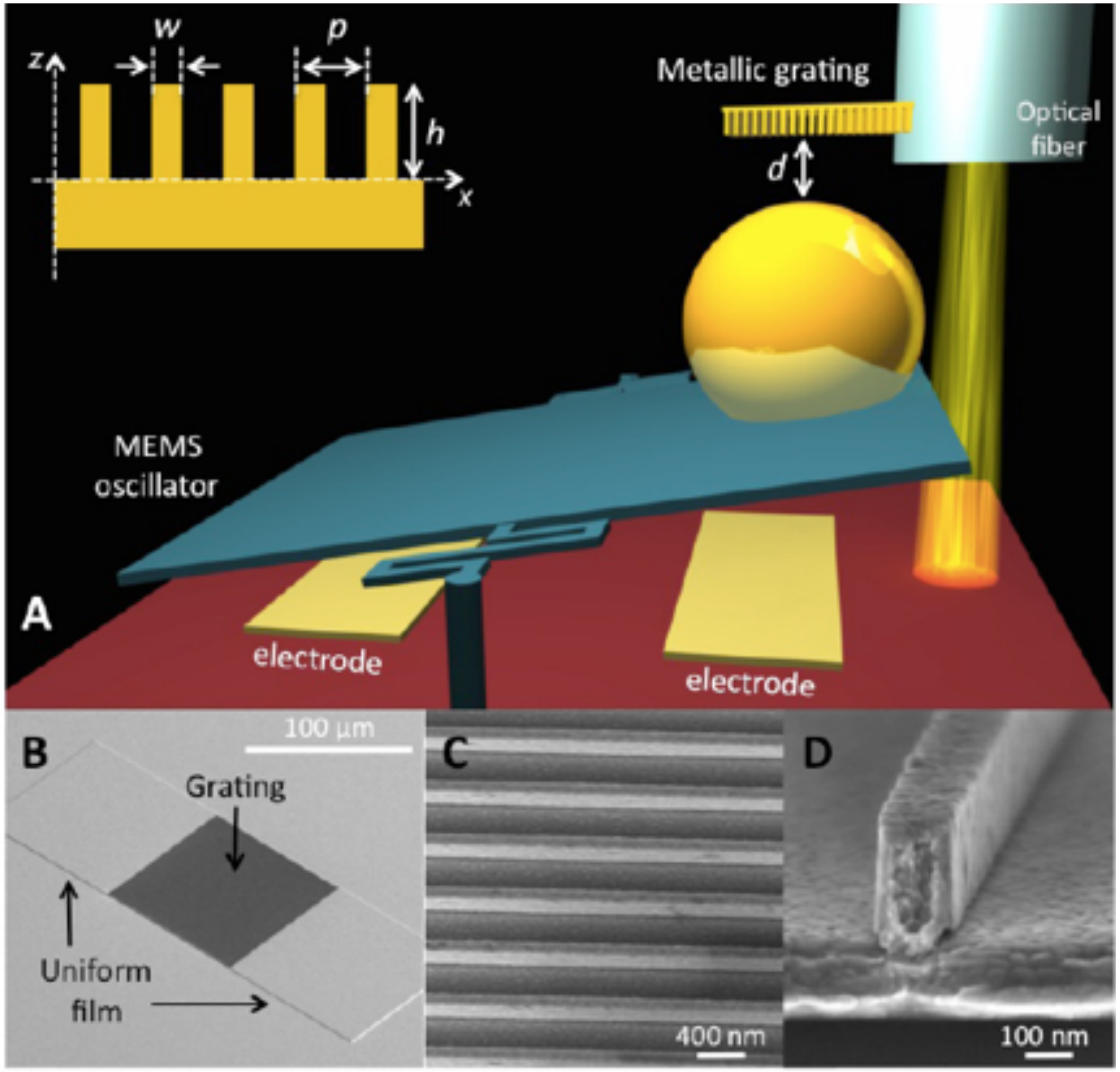
Plasmonic-photonic contributions



- At short distance the plasmonic contribution dominates and is attractive
- At large distance the two contributions are opposite in sign and balance

Can one control the Casimir force by changing the balance of the two contributions?

Grating nanostructures



Experimental set-up

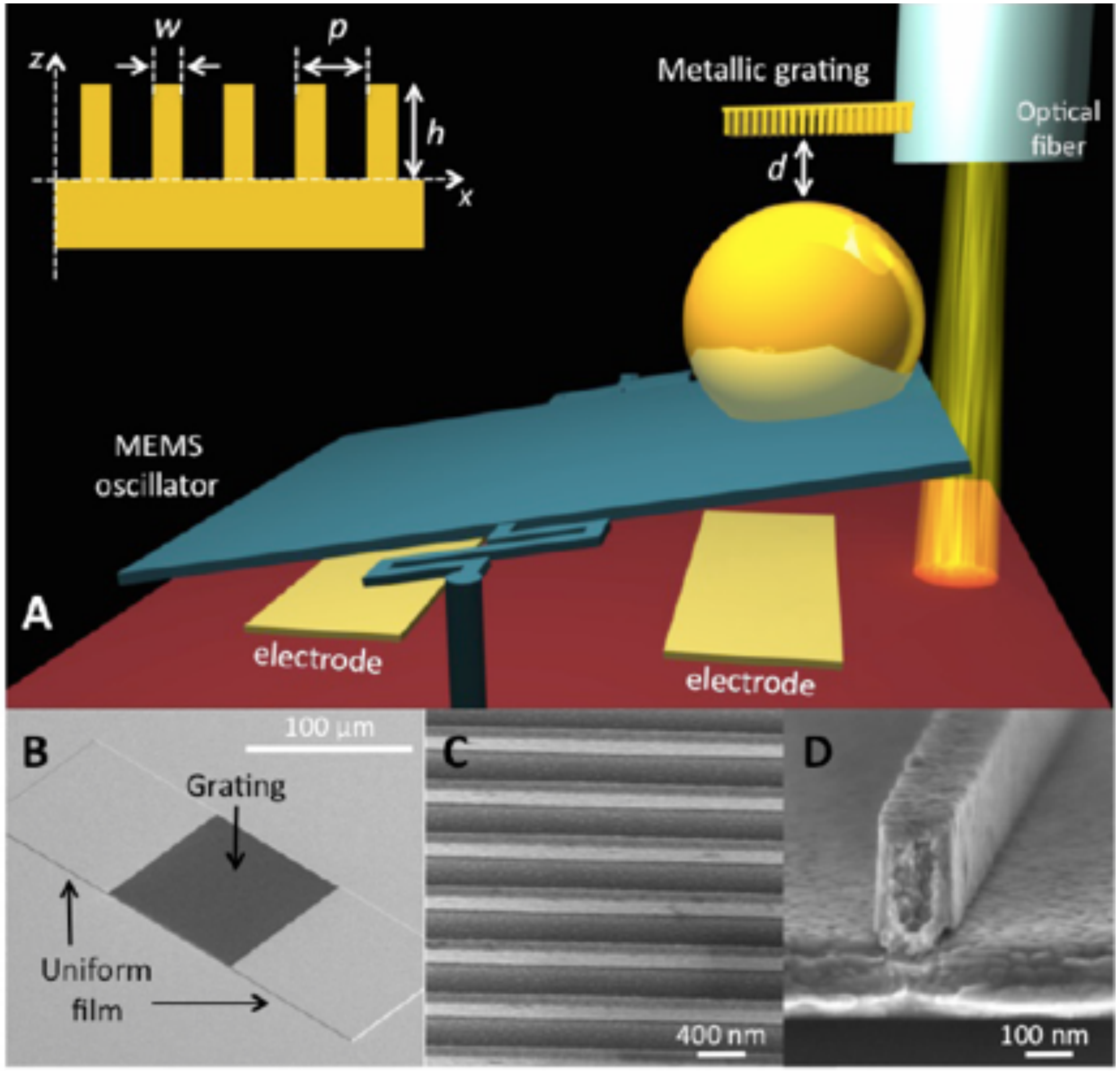
- Torsional balance set-up
- Metallic sphere

$$R = 150\ \mu\text{m}$$

- Metallic nano gratings

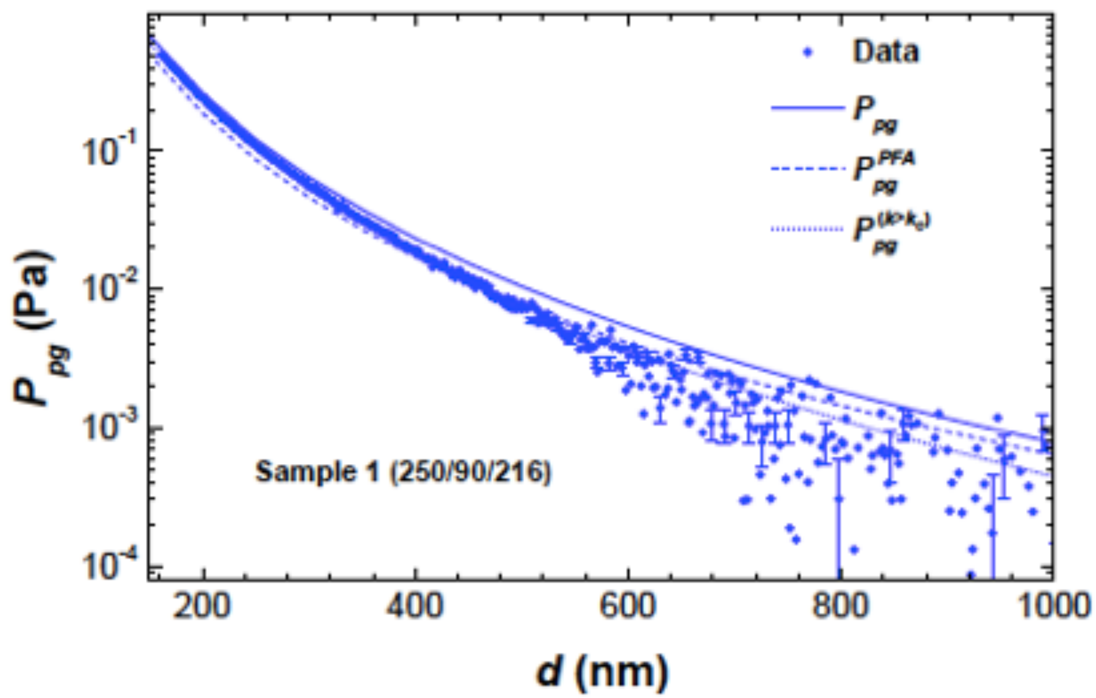
$$w, p, h \approx 100\ \text{nm}$$

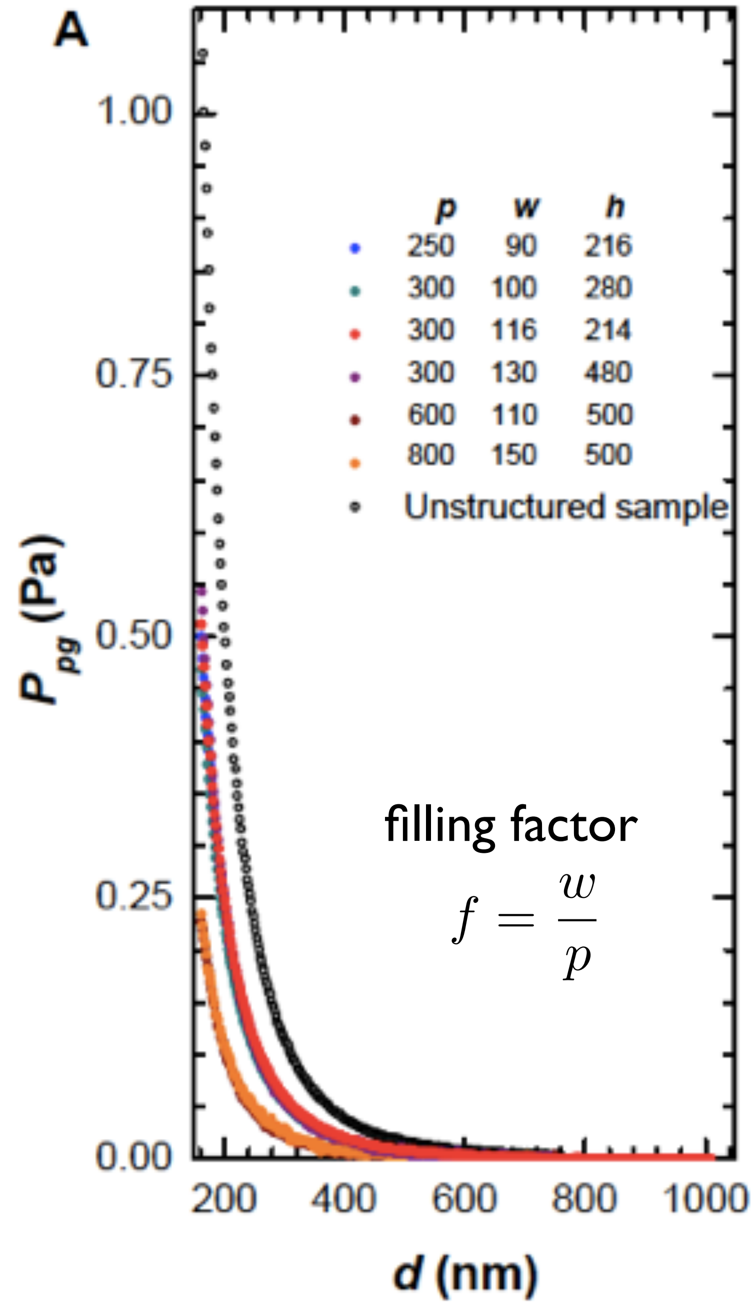
- Sputtering and electroplating



Strong force reduction

- Gratings with similar filling factor have similar force reduction
- Strong force reduction with respect to the standard plane-sphere geometry
- Results independent of fab method





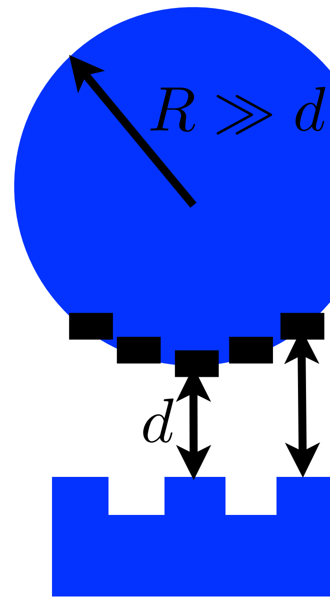
$$f = \frac{w}{p}$$

Modeling and simulation

- Use of standard PFA to treat the sphere's curvature

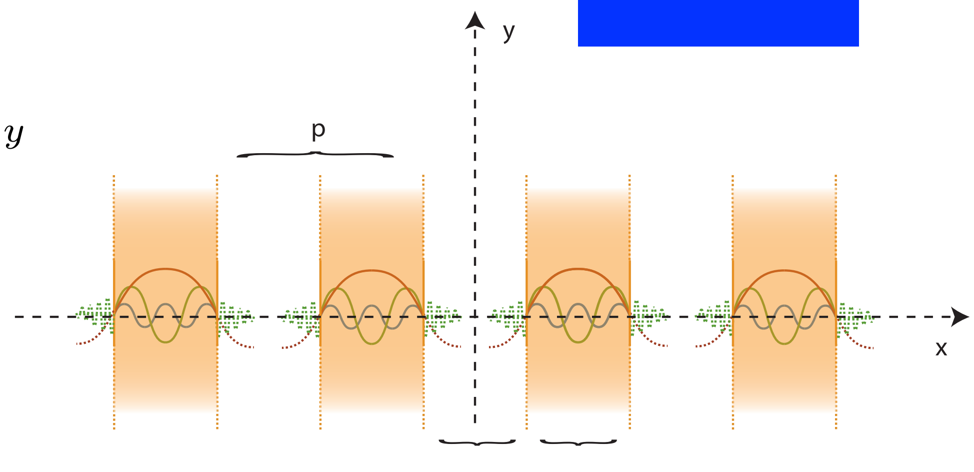
$$F'_{sg} \approx 2\pi R P_{pg} \qquad d/R < 6 \times 10^{-3}$$

- Exact computation of the plane-grating pressure P_{pg}



Scattering approach + modal expansions Li (1993)

$$\begin{pmatrix} E_z(x,y) \\ E_x(x,y) \\ H_z(x,y) \\ H_x(x,y) \end{pmatrix}_i = \sum_{\nu,s} A_\nu^{(s,i)} \mathbf{Y}^{(s,i)}[x, \eta_\nu^{(s,i)}] e^{\imath\lambda[\eta_\nu^{(s,i)}]y}$$



Analytical expressions for eigenvectors

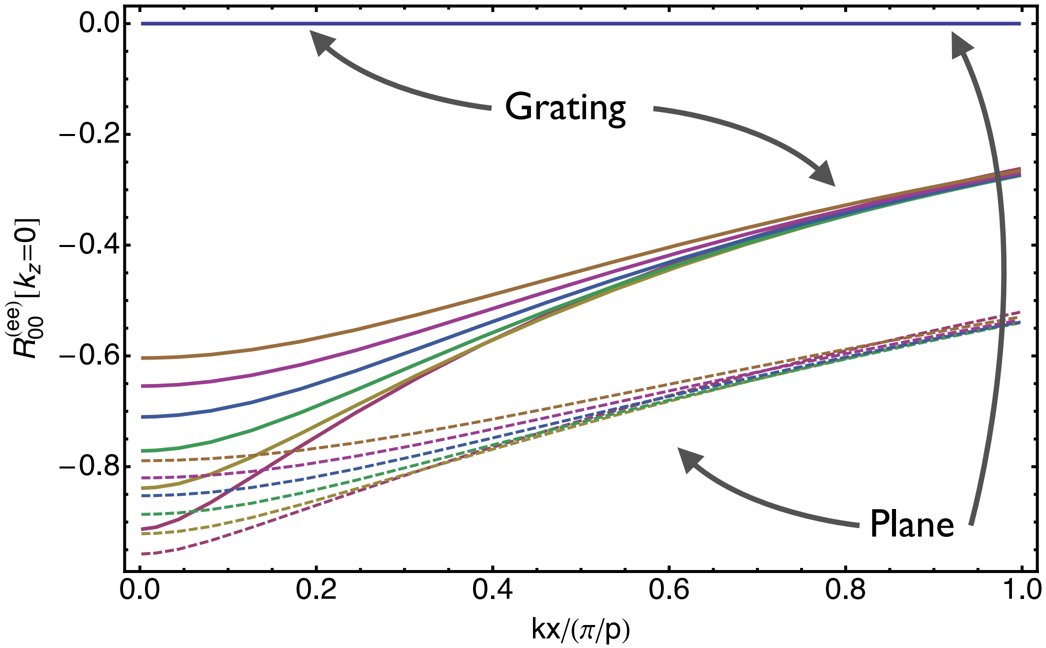
Transcendental equation for eigenvalues

$$\begin{aligned} 0 = \tilde{D}^{(s)}(\eta) &= -\cos(\alpha_0 p) + \cos(p_1\sqrt{\eta})\cos(p_2\sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}) \\ &-\frac{1}{2}\left(\frac{\sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}}{\sigma_2^{(s)}(i\xi)\sqrt{\eta}} + \frac{\sigma_2^{(s)}(i\xi)\sqrt{\eta}}{\sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}}\right)\sin(p_1\sqrt{\eta})\sin(p_2\sqrt{\eta - [\epsilon(i\xi) - 1]\xi^2}), \end{aligned}$$

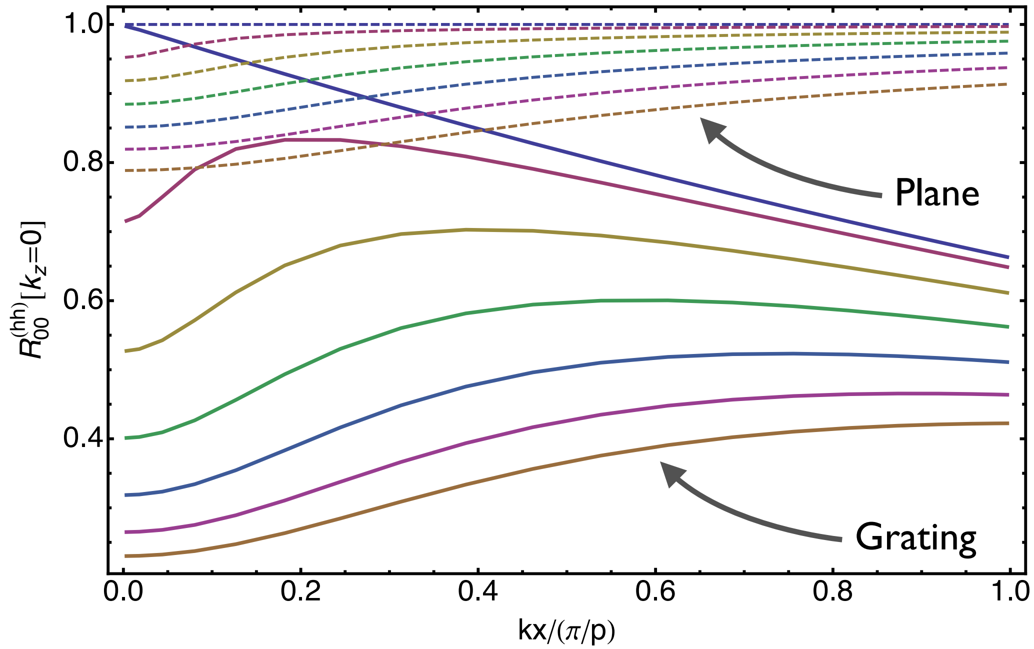
Intravaia *et al.*, PRA **86**, 042101 (2012)

Reflection matrices

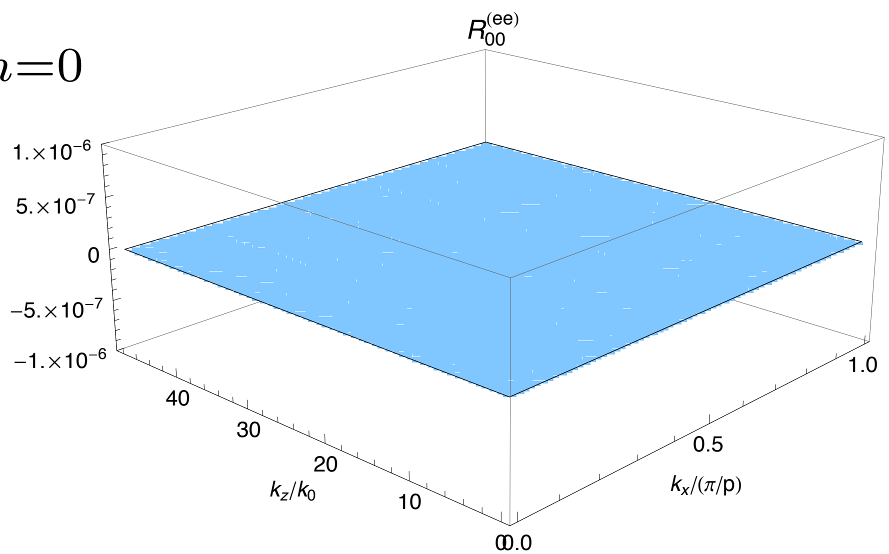
e-polarization (first 7 ξ_n's)



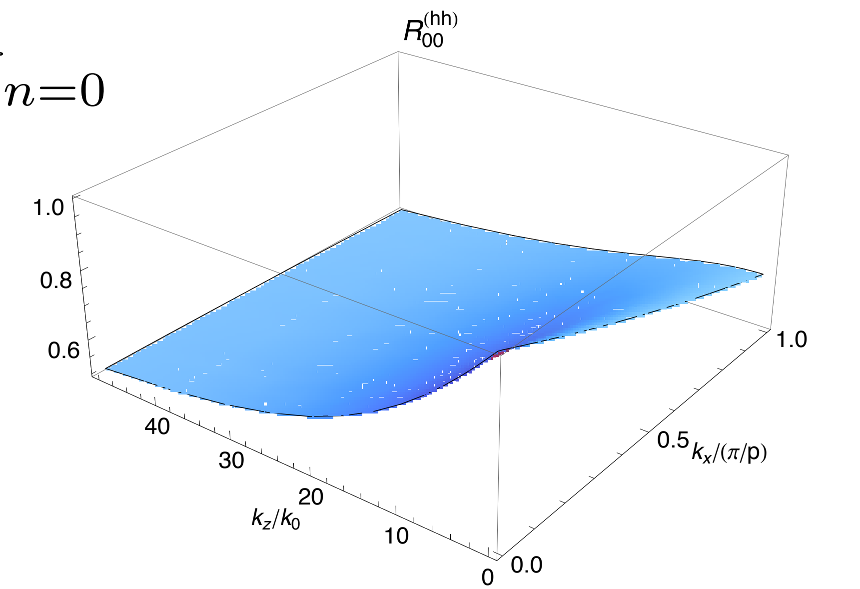
h-polarization (first 7 ξ_n's)



$\xi_{n=0}$

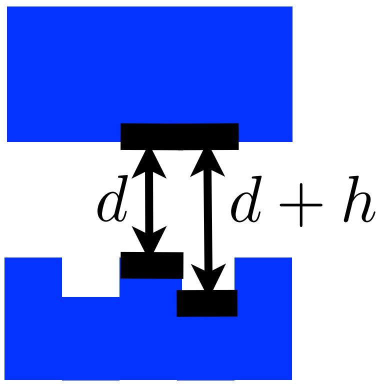


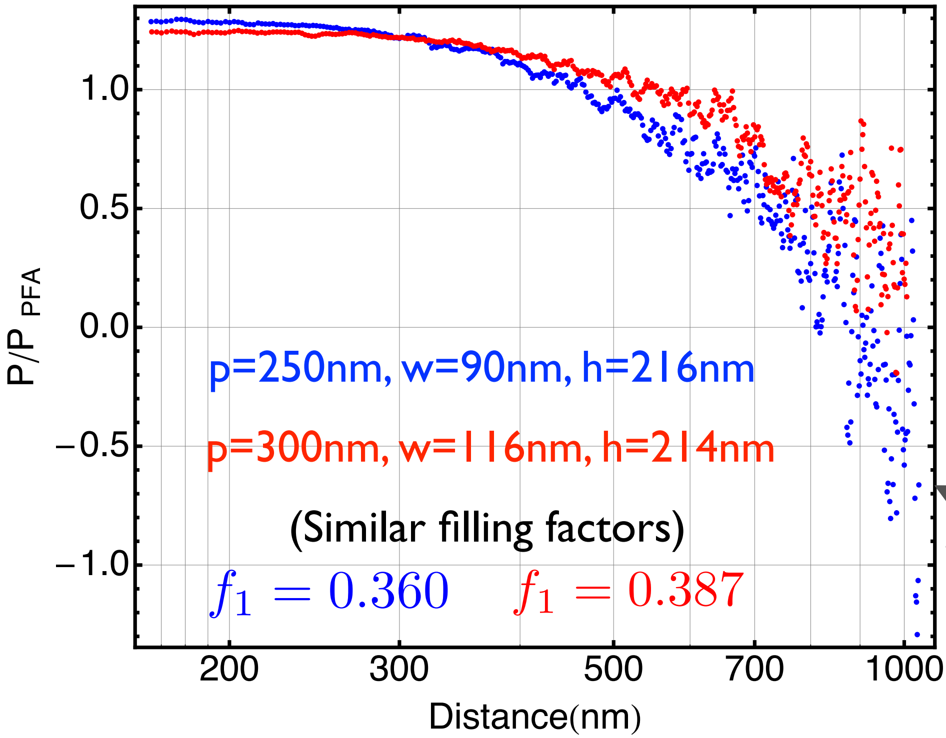
$\xi_{n=0}$



Normalizing to PFA for grating

$$P_{pg}^{\mathrm{PFA}}(d) = f P_{pp}(d) + (1-f) P_{pp}(d+h)$$





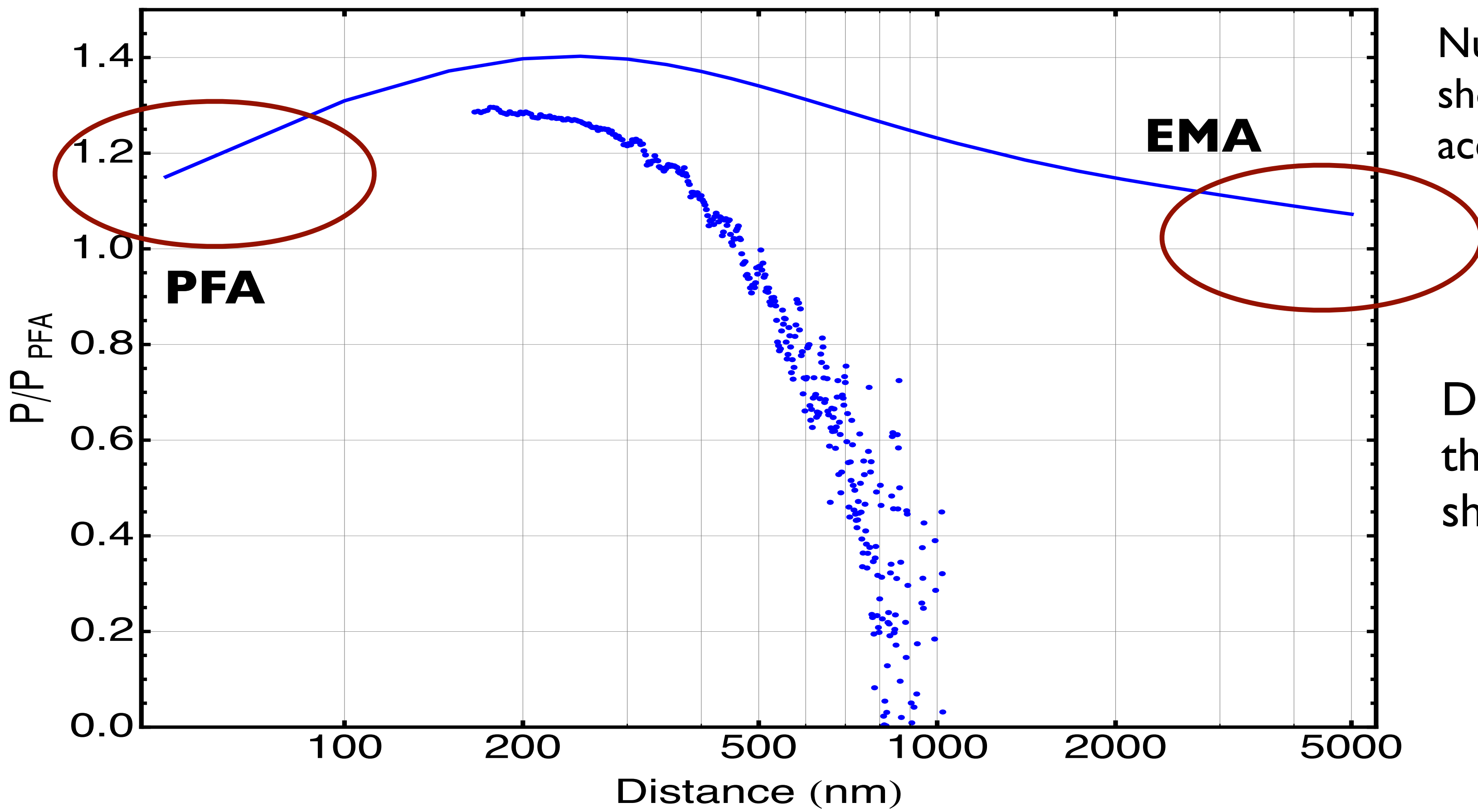
Small separations: *PFA underestimates the total pressure*.

Large separations: *PFA overestimates the exact pressure*.

Pressure is going to zero faster than d^{-4}

- Strong suppression of the Casimir force

Open problem



Numerical crosschecks show that the theory is accurate within few %

Double checks on the experiment show no apparent

Experiment/theory discrepancy: open problem in Casimir physics

nature COMMUNICATIONS

ARTICLE

Received 5 Feb 2013 | Accepted 28 Aug 2013 | Published 27 Sep 2013 DOI: 10.1038/ncomms3515 OPEN

Strong Casimir force reduction through metallic surface nanostructuring

Francesco Intravaia[1], Stephan Koev[2,3], Il Woong Jung[4], A. Alec Talin[2], Paul S. Davids[5], Ricardo S. Decca[6], Vladimir A. Aksyuk[2], Diego A.R. Dalvit[1] & Daniel López[4]

Final comments

- Quantum vacuum fluctuations induce macroscopic effects

- Casimir forces are one example of fluctuation-induced interactions

- Can be tailored by geometry, material composition, and temperature

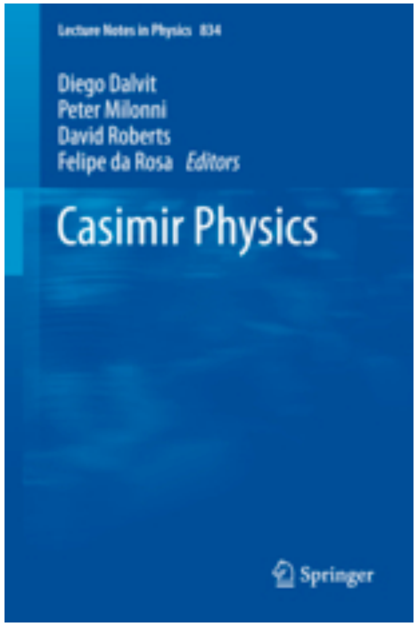
  - Observation of thermal corrections to the Casimir force

  - Strong Casimir force reduction using metallic nano-gratings

- There are still open problems in Casimir physics, e.g. how to obtain measurable force repulsion between vacuum-separated objects.

Collaborators

- Felipe da Rosa, Francesco Intravaia, Ryan Behunin, Yong Zeng, Wang-Kong Tse, Nicola Bartolo (LANL postdocs/students)
- Peter Milonni (LANL)
- Roberto Onofrio (Dartmouth)
- Steve Lamoreaux (Yale)
- Steven Johnson (MIT)
- Ricardo Decca (Indiana)
- Daniel Lopez (Argonne)
- Vladimir Aksyuk (NIST)
- Paul Davids (Sandia)
- Serge Reynaud (ENS Paris)


Lecture Notes in Physics 834
Diego Dalvit
Peter Milonni
David Roberts
Felipe da Rosa Editors
Casimir Physics
Springer

Thank you